MEASI INSTITUTE OF MANAGEMENT
CHENNAI-14
Approved by All India Council of Technical Education and
Affiliated to the University of Madras, ISO 9001:2015 Certified Institute

APPLIED OPERATIONS RESEARCH COURSE MATERIAL

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VISION & MISSION STATEMENTS

VISION;

- To emerge as the most preferred Business School with Global recognition by producing most competent ethical managers, entrepreneurs and researchers through quality education.

MISSION;

- **Knowledge through quality teaching learning process**; To enable the students to meet the challenges of the fast challenging global business environment through quality teaching learning process.
- **Managerial Competencies with Industry institute interface**; To impart conceptual and practical skills for meeting managerial competencies required in competitive environment with the help of effective industry institute interface.
- **Continuous Improvement with the state of art infrastructure facilities**; To aid the students in achieving their full potential by enhancing their learning experience with the state of art infrastructure and facilities.
- **Values and Ethics;** To inculcate value based education through professional ethics, human values and societal responsibilities.

PROGRAMME EDUCATIONAL OBJECTIVES (PEOs)

PEO 1; Placement; To equip the students with requisite knowledge skills and right attitude necessary to get placed as efficient managers in corporate companies.

PEO 2; Entrepreneur; To create effective entrepreneurs by enhancing their critical thinking, problem solving and decision-making skill.

PEO 3; Research and Development; To make sustained efforts for holistic development of the students by encouraging them towards research and development.

PEO4; Contribution to Society; To produce proficient professionals with strong integrity to contribute to society.

Program Outcome;

PO1; Problem Solving Skill; Apply knowledge of management theories and practices to solve business problems.

PO2; Decision Making Skill; Foster analytical and critical thinking abilities for data-based decision making.

PO3; Ethical Value; Ability to develop value based leadership ability.

PO4; Communication Skill; Ability to understand, analyze and communicate global, economic, legal and ethical aspects of business.

PO5; Individual and Leadership Skill; Ability to lead themselves and others in the achievement of organizational goals, contributing effectively to a team environment.

PO6; Employability Skill; Foster and enhance employability skills through subject knowledge.

PO7; Entrepreneurial Skill; Equipped with skills and competencies to become an entrepreneur.

PO8; Contribution to community; Succeed in career endeavors and contribute significantly to the community.

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Subject Code	Subject Name	L	T	P	S	C
PMF2H	APPLIED OPERATIONS RESEARCH	3	1	0	1	4

Course Objectives	
C1	To provide the students with introduction on OR and its models to aid in understanding its applicability in the various functional areas of management.
C2	To understand the concept of linear programming models in determining profit maximization and cost minimization
C3	To learn about various methods adopted in transportation and Assignments models.
C4	To determine about inventory models, replacement models, job sequencing, networking model and Queuing model
C5	To throw light on game models and the application of pure and mixed strategies in competitive environment.

SYLLABUS

Unit. No.	Details	Hours
Unit I	**Overview of operations research** – Origin – Nature, Scope & characteristics of OR – Models in OR – Application of operations research in functional areas of management.	08
Unit II	**Linear programming problem model** – Formulation – Maximization & Minimization problem - Graphical method – Simplex method – Artificial variable –- Primal & Dual.	12
Unit III	**Transportation problem:** North / West corner Solution - Stepping stone method - Vogel's approximation method - MODI method – Degeneracy - Imbalance matrix. Assignments model; Hungarian method - Traveling salesmen problem.	12
Unit IV	**Deterministic Inventory models** – Purchasing & Manufacturing models – Probabilistic inventory models - Replacement model – Sequencing - Brief Introduction to Queuing models. Networking - Programme Evaluation and Review Technique (PERT) and Critical Path Method (CPM) for Project Scheduling.	20
Unit V	**Game Theory and Strategies** –Mixed Strategies for games without saddle points - Two person zero sum games – Graphical and L.P Solutions.	08
	Total Hours	60

Reference Books

1.	Anderson, D.R., Sweeney, D.J., Williams, T.A. and Martin, K., An Introduction to Management Science; Quantitative Approach to Decision Making, 13th Edition, South Western, 2012.
2.	Gupta, P.K., and Comboj, Introduction to Operations Research, S. Chand, 2012
3.	Hiller, F., Liebermann, Nag and Basu, Introduction to Operations Research, 9th Edition, Tata McGraw-Hill Publishing Co. Ltd., 2011.
4.	Khanna, R.B., Quantitative Techniques for Managerial Decision Making, 2nd Edition,

	PHI Learning Pvt. Ltd., 2012.		
5.	Taha, H.A., Operations Research; An Introduction, 8th Edition, Pearson, 2011.		
E-Sources			
1.	http://www.pondiuni.edu.in/storage/dde/downloads/mbaii_qt.pdf		
2.	https://faculty.psau.edu.sa/filedownload/doc-6-pdf14b14198b6e26157b7eba06b390ab763-original.pdf		
3.	http://164.100.133.129;81/econtent/Uploads/Operations_Research.pdf		
4.	https://hvtc.edu.vn/Portals/0/files/636076312329739612Businessapplicationsofoperationsresearch.pdf		
5.	https://santini.in/files/slides/aua-slides.pdf		
6.	www.cbom.atozmath.com		
Assessment Tools Used			
1.	Assignments	6.	Group Discussion
2.	Internal Assessment Tests	7.	Simulation
3.	Model Exam	8.	Videos
4.	Seminars	9.	Synetics
5.	Case studies	10.	Quiz
Content Beyond Syllabus			
1.	Crew Assignments model – Dynamic programming		
2.	Simulation – Group Replacement		
3.	Sensitivity analysis –Integer programming		
Additional Reference Books			
1.	Vohra, N.D., Quantitative Techniques in Management, 4th Edition, Tata McGraw Hill Education Pvt. Ltd., 2010.		
2.	G. Srinivasan, Operations Research – Principles and Applications, PHI, 2007.		
3.	Kalavathy S, Operations Research, Fourth Edition, Vikas Publishing House, 2012		

Course Outcomes

CO No.	On completion of this course successfully, the students will;	Program Outcomes (PO)
C202.1	Obtain insight on the origin and nature of OR and also the application of various models of OR.	PO4, PO6
C202.2	Learn about the graphical, Simplex, Big M and dual methods of Linear programming problem.	PO1, PO2, PO6, PO7
C202.3	Will be well versed with the concept of transportation and Assignments models	PO1, PO2, PO6, PO7
C202.4	Have better understanding on inventory models, replacement models, job sequencing, networking model and Queuing model	PO1, PO2, PO6, PO7
C202.5	Be imparted knowledge on the various methods of game model.	PO2, PO7

Unit – I
INTRODUCTION

1. **What is Operations Research?**

Operations Research is applied decision theory. It uses any scientific, mathematical or logical means to attempt to cope with the problems that confront the executive when he tries to achieve a though going rationality in dealing with decision problems – Miller and Starr.

Operations research rather simply defined, is the research of operations. A operation is a set of acts required for the achievement of the desired outcome. Such complex interrelated can be performed by four types of systems; Man, Machine, Man-Machine unit and any organization of Men, Machines and Man-Machine units, Operation research is concerned with the last type of system.

2. **What are the characteristics of OR?**
   - Its system (or executive) orientation
   - The use of inter-disciplinary teams
   - Application of scientific method
   - Uncovering of new problems
   - Improvement in the quality of decisions
   - Use of Computer
   - Quantitative Solutions and
   - Human Factors

3. **Discuss the Limitations of Operations Research.**
   - Mathematical models, which are the essence of OR do not take into account qualitative factors or emotional factors which are quite real. All influencing factors which cannot be quantified find no place in mathematical models.
   - Mathematical models are applicable to only specific categories of problems.
   - OR tries to find optimal solution taking all the factors of the problem into account. Present day problems involve numerous such factors; expressing them in quantity and establishing relations among them requires huge calculations.
   - Being a new field, generally there is a resistance from the employees to the new proposals.
   - Management, who has to implement the advised proposals, may itself offer a lot resistance due to conventional thinking.
   - Young enthusiasts, overtaken by it's advantages and exactness, generally forget that OR is meant for men and not that men are meant for it.

4. **Enumerate with Brief Description some of the techniques of OR.**

- Inventory Models- It deals with idle resources such as men, machine money and materials. These models are concerned with two decisions ; <i> how much to order to replenish the inventory of an item and <ii> when to order so as to minimize the total cost.
- Allocation Models- Are used to solve the problems in which <i> there are number of jobs to be performed and there are alternative ways of doing them and <ii> resources or facilities are limited.

- Sequencing Models- These are applicable in situations in which has the effectiveness measure "(time cost or distance is function of the order or sequence of performing a series of job (tasks). The selection of the appropriate order in which waiting customers may be served is called sequencing.

- Project Scheduling by PERT and CPM- In large and complex project involving a number of interrelated activities, requiring a number of men, machines and materials. It is not possible for the management to make and execute an optimum schedule just by intuition. Management are, thus planning, scheduling and controlling the project PERT and CPM are two of many project management techniques used for these purpose.
- Queuing Models- It involve in the arrival of units to be served at one or more service facilities. These units (or customers) may be trucks arriving a loading station, customers entering a departmental store, etc. The arriving units may form one line and get serviced. This will occur when the system has a single service channel. The system may have a number of service channels, which may be arranged in parallel or in a series or a complex of both.

5. **Explain briefly the general methods for solving OR models.**
   - Analytic Procedure: Solving models by classical mathematical techniques like differential calculus, finite differences and so on, to obtain analytic solutions.
   - Iterative Procedure: Starts with a trial solution and a set of rules for improving it by repeating the procedure until further improvement is not possible.
   - Monte-Carlo technique: Taking sample observations, computing probability distributions for the variable using random numbers and constructing some functions to determine the values of the decision variables.

6. **Explain briefly the different phases of operations research.**

   - Observe the Problem Environment - The activities that constitute this step area visits, conferences, etc. With the sufficient information obtained from the activities, the OR scientist us better prepared to formulate the problem.
   - Analyze and define the Problem- In this not only the problem is defined, but also uses, objectives and limitation of the study are stressed in the light of the problem. The end result is a clear grasp of need for a solution and understanding it's nature.
   - Develop a Model- To construct a model. A model is representation of some real or abstract situation. Operations research models are basically mathematical models representing systems, process or environment in form of equations, relationships or formulae.

- Select the appropriate Data Input- No model will work properly if data input is not appropriate. Hence tapping the right kind of data is a vital step in OR process. Important activities in this step are analyzing internal, external data and facts.
- Provide a solution and test reasonableness- Here is to get a solution with the help of a model and data input. Such a solution is not implemented immediately. First is to test the model and find the limitations, if any.
- Implement the Solution- Implementation of the solution obtained in previous step is the last step in OR process. Implementation involves so many behavioral issues.

7. **Discuss the importance of operations research in decision-making process.**
   - Better Decisions- Operations Research models frequently yield actions that do improve in intuitive decision-making. A situation may be complex so that the human mind can never hope to assimilate all the significant factors without the aid of OR guided computer analysis.
   - Better Co-ordinations- Sometimes operations research has been instrumental in bringing order out of chaos. For instance, an OR oriented planning model becomes a vehicle for co-ordinating marketing decisions within the limitations imposed on manufacturing capabilities.
   - Better Control- The managements of large organization recognize that it is extremely costly to require continuous executive supervision over routine decisions.
   - Better System- Often, an OR study is initiated to analyse a particular decisions problem, such as whether to open a new warehouse. Afterwards the approach is further developed in to a system to employed repeatedly. Thus, the cost of undertaking the first application may proluce benefits.

8. **Discuss the significance and scope of operation research in modern management.**

Significance of OR in modern management

1. Marketing Management-
   (a) product selection
   (b) competitive strategies
   (c) advertising strategy

2. Production Management
   (a) Production scheduling
   (b) Project scheduling
   (c) Allocation of resources location of factories and their sizes
   (d) Equipment replacement and maintenance
   (e) Inventory policy

3. Finance Management-
   (a) cash flow analysis
   (b) capital requirement
   (c) credit policies
   (d) credit risks

4. Personal Management-
   (a) recruitment policies
   (b) assignment of jobs
5. Purchasing and procurement-
   (a) rules of purchasing
   (b) determining the quality
   (c) determining time of purchases
6. Distribution-
   (a) location of warehouses
   (b) size of the warehouses
   (c) rental outlets
   (d) transportation strategies.

Scope of OR

1. Accounting-
   Cash flow planning
   Credit policy analysis
   Planning of delinquent account strategy

2. Construction-
   Allocation of resources to projects
   Determination of proper workforce
   Deployment of workforce
   Project scheduling, monitoring and control

3. Facilities planning-
   Factory size and location decision
   Hospital planning
   International logistics system design
   Estimation of number of facilities required
   Transportation loading and unloading
   Warehouse location decision

4. Finance-
   Dividend policy making
   Investment analysis
   Portfolio analysis
5. Manufacturing-
   Inventory control
   Projection marketing balance
   Projection scheduling
   Production smoothing

6. Marketing-
   Advertising budget allocation
   Product introduction timing
   Selection of product mix

7. Organizational behaviour-
   Personal justification/planning
   Scheduling of training programmes
   Skills balancing
   Recruitment of employees

8. Purchasing
   Material transfer
   Optimal buying
   Optimal reordering

9. Research and development
   Control of R&D projects
   Product introduction planning

9. Explain various types of OR models and indicate their application to productions, inventory and distribution systems.

Types of OR models;

1. **Iconic (physical) model**
   This is pictorial representation of various aspects of a system
2. **Analogue or Schematic model**
   This uses one set of properties to represent another set of properties which a system under study has.
3. **Mathematical or symbolic model**
   This uses a set of mathematical symbols (letters, numbers etc.) top represent the decision variable of a system under consideration. These variables are related by mathematical equations or in equations which describe the properties of the system.
4. **Static model**
   This is a model which does not take time into account. It assumes that the values of the variables do not change with time during a certain period of time horizon.
5. **Dynamic model**
   This model considers time as one of important variables
6. **Deterministic Model**
   This model which doesn't take uncertainty into account.
7. **Stochastic model**
   This model which considers uncertainty as an important aspect of the problem.

8. **Descriptive model**
   This is one which describes a situation or system.
9. **Predictive model**
   This is one which predicts something based on data.
10. **Prescriptive model**
    This is one which prescribes or suggests a course of action for a problem.
11. **Analytic model**
    This is one which exact solution is obtained by mathematical methods of closed form.
12. **Simulation Model**
    This is a representation of reality through the use of a model or device which will react in the same manner as reality under given set of conditions. Once a simulation model is designed, it takes only a little time, in general, to run a simulation on a computer.

10. State the principles of modeling?

- Do not build up a complicated model when a simple one would surface
- Beware of moulding the problems to fit a technique
- Deductions must be made carefully.
- Models should be validated prior to implementation
- A model should neither be pressed to donor criticized for falling to do that for which it was never intended.,
- Beware of overselling the model in cases where assumptions made for the construction of the model can be challenged.
- The solution of the model cannot be more accurate than the accuracy of the information that goes into the construction
- Models are only aids in decision-making
- Models should not be complicated. It should be as simple as possible
- Models should be as accurate as possible

Unit – II
LINEAR PROGRAMMING

What is linear programming?

Linear programming deals with the optimization of a function of variables known as objective function subject to a set of linear equations and for inequality known as constraints. The objective function may be profit, cost, production capacity or any other measure of effectiveness, which may of profit, cost, production capacity. The constraints may be imposed by different resource such as market demand production process and equipment, storage capacity raw materials availability, etc. By linearity it means a mathematical expression in which the expression among the variables are linear e.g. the expression a1x1+a2x2+a3x3+…..+anxn in linear.

It was in 1947, the George Datzing and its associates found out the techniques for solving military planning problem while they were working on a project for U.S.Air force. This techniques consists of representing the various activities of an organization as a linear program model and arriving to the optimal programmer by minimizing a linear objective functions.

REQUIREMENT FOR A LINEAR PROGRAMMING PROBLEM

All organization have at their disposal, men, machine, money and materials, the supply of which may be limited. If the supply of these resource were unlimited, the need for management tools like linear programming would not arise to all.

Generally speaking, linear programming can be used for optimizing problems if the following conditions are satisfied:

- There must be a well defined objective function which is to be either maximized or minimized and which can be exposed as a linear function of decision variables.
- There must be a constraints on the mount of extent of attainment of the objective and these constraints must being expressed as linear equations.
- There must be alternative source of action.
- Another necessary requirement is the decision variables should be interrelated and non-negative. The non-negativity condition shown the linear programming deals with real life situations for which negative equations are generally illogical.
- As stated earlier the resource must be in limited supply.

ADVANTAGES OF LINEAR PROGRAMMING METHOD:

- It helps in attaining the optimum use of productive factors for example more efficient of man power and machine can be obtained by the use of linear programming.
- It improves the quality of decision. The individual having a clear picture of a relationship within the same equations, inequalities or constraints can have a better idea about the problem.
- It also helps in providing the letter tools for adjustments to meet the changing condition.
- Most business problem involve constraint like raw material availability, market demand, etc. linear programming handled such situation, since it allows modification of its mathematical solution.

- It highlights the bottleneck in the production. When bottleneck occurs, some machines cannot meet demand while others remain idle, at least part of the time.

LIMITATIONS OF LINEAR PROGRAMMING MODEL:

- For large problems having limitations and constraints, the computational difficulties are enamors, even when assistance of large digital computers are available.
- It may yield functional valued assures for the decisions variables, whereas it may happen that only integer values of the variables are logical.
- It is applicable to only static situation since it does not take into account the effect of time.
- It assumes that the value of the co-efficient of the decision variables in the objective function as well as in all the constrains as known with certainty.
- In some situation it is not possible to express both the objective function and constrain in linear form.
- Linear programming deals with problem that have the single objective one has to apply good programming under such situation.
- When comparison is made between advantage and disadvantages, its advantages clearly over weight its limitation. Like other mathematical tools only help manager to take better decision, they are no way a substitute for the manager.

THEORY OF SIMPLEX METHOD:

Simplex method also called simplex technique or simplex algorithm was developed in 1947 by G.B.Dantzing, an American mathematician. It has the advantage of being universal i.e., any linear model for which the solution exists, can be solved by it. In principle, it consists of starting with a certain solution of which all that we know is that it is basic feasible i.e., it satisfies the constraints as well as non-negativity conditions. We then, improve upon this solution at consecutive stages, until, after a certain finite number of stages, we arrive at the optimal solution.

The simplex method provides an algorithm which consists in moving from one vertex region of feasible solutions to another in such a manner that the value of the objective function at the succeeding vertex is less in a minimization problem (or more in a maximization problem) than at the preceding vertex. This procedure of jumping from one vertex to another is then repeated. Since the number of vertices is finite, this method leads to an optimal vertex to another is then number of steps.

The basis of the simplex method consists of two fundamental conditions.

<u>The feasibility condition:</u>

It ensures that if the starting solution is basic feasible, only basic feasible solution will be obtained during computation.

<u>The optimality condition:</u>

it guarantees that only better solution(as compared to the current solution)will be encountered.

Simplex method makes use of the following three points in achieving a systematic reduction from an infinite number of solution to a finite number of promising solution.

1. if there are m equating constraints and m + n is the number of variable(m<=n), a start for the optimal solution is mades by putting n unknowns(variable)(out of(m+n)unknowns)equal to zero and then solving for the m equation in remaining m unknowns, provided that the solution exit; and is unique. the n zero variable called non-basic variable and the remaining m variable are called basic variable, which form a basic solution.

2. It is known that in linear programming problem, variable must be non-negative, since the basic solution selected by condition 1 above are not necessarily non-negative, the number of alternatives can be further reduced by eliminating all infeasible basic solution. a condition called feasibility condition is then provided which insure that the next basic solution to be selected from all the possible basic solution is always feasible. this is called feasible solution. The basic variable set equal to zero is called a "leaning variable" while the new one is called an "entering variable".

3. The entering variable can be so selected that it improves the value of objective function so that the new solution is better then previous one. this is achieved by the use of other condition called optimality condition which selects the entering variable which produces the largest per unit gain in the objective function. this procedure is repeated successively until no further improvement in the value of the objective function is possible. The final solution is then called an optimal basic feasible solution or simply optimal solution.

Artificial variable techniques:-

The last section death with LPP's with less than or equal [<=] type constraints. this property together with the fact that the right hand side constraints is non-negative, provides us with a ready starting basic feasible solution that comprises of all slack variables. There are however many liner programming problems for which mere slack variable cannot provide such solution. For those, we accomplish by adding artificial variable which play the role of slack variables. However since such artificial variable have no physical meaning in the original model (hence the name artificial), provisions must be made to make them zero level at the optimum iteration. in other words, we use them to start the solution, and abandon them once their work has been over. A logical way to achieve this objective is to penalize the artificial variable in the objective function. Two (closely related) networks based on the use of penalties are available to solve this type of LLP.they are:
1. Big "M" method,
2. Two-phase simplex method.

The BIG 'M' METHOD:-

STEP 1: Convert the LLP into express it in the standard form by introducing slack and/or surplus variable as the case maybe.
STEP 2: Introduce non-negatives variables to the left hand sides of all the constraints of (>=or=) type. These variables are called artificial variable. The purpose of introducing artificial variables is just to obtain an

initial basic feasible solution. In order to get rid of the artificial variables in the final optimum iteration, we assign a very large penalty-'-M' to these artificial variables in the objective function for maximization objective function.
STEP 3: Solve the modified liner programming problem by simplex method, whenever an artificial variable happens to leaves basic, we drop that artificial variable and omit all the entries corresponding to its column from the simplex table.
STEP 4: Application of simplex method is continued until either an optimum basic feasible solution is obtained or there is an indication of the existence of an unbounded solution to the given LLP.

The Two-Phase Simplex Method:

The two-phase simplex method is another method to solve an LPP in which some artificial variable are involved. The solution can be obtained in two-phase.

PHASE 1: It consists of the following steps:

Step 1: It convert the LPP into maximization form and insure that all b;(constants term) are non-negative. If some of them are negative, make them non-negative by multiplying both sides of those in equations by -1.

Step 2: Add artificial variable a;(>=0) to the LHS of constraint of (= and <=)to complete the identity matrix I.
Step3: Express the given LPP in standard form.
Step4: Obtain and initial basic feasible solution.
Step5: Assign a cost -1 to each artificial variable and a cost o to all other variable(in place of their original cost)in the objective function ,the new objective function is Max, z*=-A1-A2-.....-An where Ai, are the artificial variable,
Step6: Write down the auxiliary LPP in which in new objective function is to be maximize subject to the given set of constraints
Step7: Solve the auxiliary LPP by simplex method until either of the following three possibilities arise.
(1) Max z*<0 and at least one artificial variable appears in the optimum basis at a +ve level. In this case no feasible solution exits stop the procedure.
(2) Max z*=o and at least one artificial variable appears in the optimum basis at zero level.
(3) Max z*=o and no artificial variable appears in the optimum basis at zero level. If case (2) and (3) arise proceed to phase 2:
PHASE-II: use the optimum basic feasible solution of phase I as a starting solution for the original LPP .Assign actual cost to the variable in the objective function and a cost 0 to every artificial variable in the basis at zero level. Delete the artificial variable column from the table which is eliminated from the basis in phase one. Applied simplex method to the modified simplex obtain at the end of the phase 1 till and optimum basis feasible solution is obtained or till the is a indication and bounded solution.

ASSUMPTIONS IN LINEAR PROGRAMMING

A linear programming is based on the following assumptions

i) Proportionality:

A basic assumption of linear programming is that proportionality exists in the objective function and the constraints. This assumptions implies that if a product yields a profit of Rs.10, the product profit earned from the sale of 12 such products will be Rs.(10*12) Rs.120.. This way may not always be true because of quantity discounts. Further even if the sale price is constant, the cost of manufacturing may vary with the no of unit produced so the profit may also vary per unit.

ii) Additivity:

it mean that if we use t1 hours of machine A to make product 1 and t2 hours to make product 2 the total time require to make product 1 and 2 on machine A is t1+t2 hours . This how ever is true only if the change over time from product 1 to product 2 is negligible .Some process may not behave in this way.
For example-when several liquids of different chemicals positions are mixed, the resulting volume may not be equal to the sum of the volumes of an individual.

iii) Continuity:

Another assumptions underlying the linear programming model is that the decision variable are continuous i.e., they are permitted to take any non-negative values that satisfy the constraints. However, there are problems where in variables are restricted to have integral values.

iv) Certainty:

Another assumptions underlying a linear programming model is that the various parameters, namely the objectives functions coefficient, RHS coefficient of the constraints and resource values in the constraints are certainty or precisely and the values do not change.

v) Finite choices:

A linear programming model also assumes that a finite (limited) number of choices (alternatives) are available to the decision maker and that the decision variable are interrelated and non-negative. The non-negative condition shows that linear programming deals with the real life situations as it is not possible to produce/use negative quantities.

APPLICATION OF LINEAR PROGRAMMING MODEL

Though in the world we leave most of the events are non linear yet there are many instances of linear events that occur in day to day life therefore an understanding of linear programming and its applications in solving problems is utmost essential for today's manager

Linear programming is widely to solve a number of business, industrial, military, economic, marketing, distribution and advertising problems. Three primary for its wide use are

1. A large number of problems from different fields can be represented or at least approximated to linear programming problems.

2. Powerful and efficient techniques for solving LP problems are available.

3. LP models can handle data variation (sensitivity analysis) easily.

However solutions procedures are generally interactive and even medium size problems require manipulation of large amount of data. But with the development of digital computers, disadvantages has been completely overcome as these computers can handle very large LP. Problems in comparatively a little time of a low cost.

AREAS OF APPLICATION OF LINEAR PROGRAMMING

Linear Programming is one of the most widely applied techniques of operations research in business industry and numerous fields. A few areas of its application are given below.

1. Industrial Applications:-

a) Product mix problems: An industrial concern has available a certain production capacity (men, money, material, market etc) on various manufacturing process to manufacture various products. Typically different products will have different selling prices will require different amounts of production. Capacity at the several processes and will therefore have different unit profits. There may also be stipulations (conditions) on maximum and or minimum product levels. This problem is to determine the product mix that will maximize the total profit.

b) Blending Problems: These problems are likely to arise when a product can be made from a variety of available raw materials of various compositions and prices. The manufacturing process involves blending (mixing) some of these materials in varying quantities to make a product of the desired specifications.

c) Production scheduling problems: They involve the determination of optimum production schedule to meet fluctuating demand. The objective is to meet demand. Keeping inventory and employment at reasonable minimum levels while minimizing the total cost of production and inventory.

d) Trim loss problems: They are applicable to paper sheet metal and glass manufacturing industries where items of standard sizes have to be cut to smaller sizes as per customer requirements with objective of minimizing the waste produced.

e) Assembly line balancing: It relates to a category of problems wherein the final product has a number of different components assembled together. These components are to be assembled in a sequence or set of sequences. Each assembly operator is to assigned the task combination of tasks so that his task time is less than or equal to the cycle time.

f) Make or Buy: They arise in an organization in the face of production capacity limitation and sudden spurt in demand of its products. The manufacturer not being sure of the demand. Pattern, is usually reluctant to add additional capacity and has to make a decision regarding the products to be manufactured with his won resources and the products to be sub-contracted. So that total cost is minimized.

2) Management Applications:

a) Media selection problems: They involve the selection of advertising mix among different advertising media such as T.V., radio magazines and newspaper that will maximize public exposure to company's product. The constraints may be on the total advertising budget maximum expenditure in each media.

b) Portfolio selection problem: They are frequently encountered by banks, financial companies, insurance companies investment services etc. a given amount is to be allocated among several investment alternatives such as bonds, saving certificates, common stock, mutual fund, real estate etc.

c) Profit Planning problems: They involve planning profits on fiscal year basis to maximize profit margin from investment in plant facilities, machinery, inventory and cash on hand.

d) Assignment Problems: They are concerned with allocation of facilities (men or machines) to jobs. Time required by cash facility to perform each jobs is given and the problems is to find optimum allocation (one job to one facility). So that the total time to perform the jobs is minimized.

e) Transportation problem: They involve transportation of products from say 'n' sources situated at different locations to say 'm' different destinations. Supply position at the sources demand at destination. Freight charges and storage cost etc are known and the problem is to design the optimum transportation plan to minimize the total transportation cost.

3) Miscellaneous Applications:

a) Diet problems: They form another important category to which linear programming has been applied. Nutrient contents such as vitamins, proteins, fats, carbohydrates, starch etc in each of food stuff known.
b) Agriculture problems: These problems are concerned with the allocation of input resources such as acres of land water labour fertilizers, and capital to various crops so as to minimum net revenue.
c) Flight scheduling problems: They are devoted to the determination of the most economical pattern and timings or flights that result in the most efficient use of aircrafts and crew.
d) Environment protection: They involve analysis of different alternatives for efficient disposal, paper recycling and energy policies.
e) Facility Location: These problems are concerned with the determination of best locations of public parks, libraries and recreation areas, hospital, ambulance, depots, telephone exchange.

PROBLEMS

Solve the following problems using Simplex Method

1. Solve the following

   Maximize $Z = 5X_1 + 3X_2$

   Subject to: $X_1 \leq 12$

   $X_1 + 2X_2 \geq 14$

   $X_2 \leq 5$

   $X_1 \geq 4$

   $X_1, X_2 \geq 0$

2. Maximize $Z = 10X_1 + X_2 + 2X_3$

   subject to the constraints:

   $X_1 + x_2 - 2X_3 \leq 10$

   $4X_1 = 20$

   $-2X_1 + 3X_3 \geq -12$

   Non – Negativity

   $X_1, X_2, X_3 \geq 0$

3. Solve to maximize $Z = 2X_1 + 3X_2$

   Subject to the constraints:

   $-X_1 - 2X_2 \leq 4$

   $X_1 + 2X_2 \leq 6$

   $X_1 + 3X_2 \leq 9$

   Non - Negativity

   $X_1, X_2, X_3 \geq 0$

4. Solve to maximize $Z = 15X_1 + 3X_2 + 2X_3$

   Subject to the constraints:

   $4x_1 + 2X_2 + X_3 \geq 20$

   $-2X_1 + 4X_3 \geq -12$

   $X_1 + X_2 - 2X_3 \leq 30$

   $X_1, X_2, X_3 \geq 0$

5. maximize $Z = 12X_1 + 2X_2 + X_3$

   Subject to the constraints:

   $X_1 + x_2 - 2X_3 \leq 20$

   $4X_1 + 2X_2 + X_3 \leq 12$

   $-2X_1 + 3X_3 \geq -12$

   $X_1, X_2, X_3 \geq 0$

6. Solve the following

   Maximize $Z = 5X_1 + 3X_2$

   Subject to: $X_1 \leq 12$

   $X_1 + 2X_2 \geq 14$

   $X_2 \leq 5$

   $X_1 \geq 4$

   $X_1, X_2 \geq 0$

7. Maximize $Z = 10X_1 + X_2 + 2X_3$

   subject to the constraints:

   $X_1 + x_2 - 2X_3 \leq 10$

   $4X_1 = 20$

   $-2X_1 + 3X_3 \geq -12$

   Non – Negativity

   $X_1, X_2, X_3 \geq 0$

8. Solve to maximize $Z = 2X_1 + 3X_2$

   Subject to the constraints:

   $-X_1 - 2X_2 \leq 4$

   $X_1 + 2X_2 \leq 6$

   $X_1 + 3X_2 \leq 9$

   Non - Negativity

   $X_1, X_2, X_3 \geq 0$

9. Solve to maximize $Z = 15X_1 + 3X_2 + 2X_3$

   Subject to the constraints:

   $4x_1 + 2X_2 + X_3 \geq 20$

$-2X_1 + 4X_3 \geq -12$

$X_1 + X_2 - 2X_3 \leq 30$

$X_1, X_2, X_3 \geq 0$

10. maximize $Z = 12X_1 + 2X_2 + X_3$

Subject to the constraints:

$X_1 + x_2 - 2X_3 \leq 20$

$4X_1 + 2X_2 + X_3 \leq 12$

$-2X_1 + 3X_3 \geq -12$

$X_1, X_2, X_3 \geq 0$

11. Solve the following

Maximize $Z = 5X_1 + 3X_2$

Subject to: $X_1 \leq 12$

$X_1 + 2X_2 \geq 14$

$X_2 \leq 5$

$X_1 \geq 4$

$X_1 X_2 \geq 0$

12. Maximize $Z = 10X_1 + X_2 + 2X_3$

subject to the constraints:

$X_1 + x_2 - 2X_3 \leq 10$

$4X_1 = 20$

$-2X_1 + 3X_3 \geq -12$

Non – Negativity

$X_1, X_2, X_3 \geq 0$

13. Solve to maximize $Z = 2X_1 + 3X_2$

Subject to the constraints:

$-X_1 - 2X_2 \leq 4$

$X_1 + 2X_2 \leq 6$

$X_1 + 3X_2 \leq 9$

Non - Negativity

$X_1, X_2, X_3 \geq 0$

14. Solve to maximize $Z = 15X_1 + 3X_2 + 2X_3$

Subject to the constraints:

$4x_1 + 2X_2 + X_3 \geq 20$

$-2X_1 + 4X_3 \geq -12$

$X_1 + X_2 - 2X_3 \leq 30$

$X_1, X_2, X_3 \geq 0$

15. maximize $Z = 12X_1 + 2X_2 + X_3$

Subject to the constraints:

$X_1 + x_2 - 2X_3 \leq 20$

$4X_1 + 2X_2 + X_3 \leq 12$

$-2X_1 + 3X_3 \geq -12$

$X_1, X_2, X_3 \geq 0$

16. Maximize $Z = 5X_1 + 3X_2$

Subject to: $X_1 \leq 12$

$X_1 + 2X_2 \geq 14$

$X_2 \leq 5$

$X \geq 4$

$X_1, X_2 \geq 0$

17. Maximize $Z = 10X_1 + X_2 + 2X_3$

subject to the constraints:

$X_1 + x_2 - 2X_3 \leq 10$

$4X_1 = 20$

$-2X_1 + 3X_3 \geq -12$

$X_1, X_2, X_3 \geq 0$

18. Solve to maximize $Z = 2X_1 + 3X_2$

Subject to the constraints:

$-X_1 - 2X_2 \leq 4$
$X_1 + 2X_2 \leq 6$
$X_1 + 3X_2 \leq 9$
$X_1, X_2, X_3 \geq 0$

19. Solve to maximize $Z = 15X_1 + 3X_2 + 2X_3$

Subject to the constraints:

$4x_1 + 2X_2 + X_3 \geq 20$
$-2X_1 + 4X_3 \geq -12$
$X_1 + X_2 - 2X_3 \leq 30$
$X_1, X_2, X_3 \geq 0$

20. maximize $Z = 12X_1 + 2X_2 + X_3$

Subject to the constraints:

$X_1 + x_2 - 2X_3 \leq 20$
$4X_1 + 2X_2 + X_3 \leq 12$
$-2X_1 + 3X_3 \geq -12$
$X_1, X_2, X_3 \geq 0$

21. Solve the following

Maximize $Z = 5X_1 + 3X_2$

Subject to: $X_1 \leq 12$
$X_1 + 2X_2 \geq 14$
$X_2 \leq 5$
$X_1 \geq 4$
$X_1, X_2 \geq 0$

22. Maximize $Z = 10X_1 + X_2 + 2X_3$

subject to the constraints:

$X_1 + x_2 - 2X_3 \leq 10$
$4X_1 = 20$
$-2X_1 + 3X_3 \geq -12$

Non – Negativity

$X_1, X_2, X_3 \geq 0$

23. Solve to maximize $Z = 2X_1 + 3X_2$

Subject to the constraints:

$-X_1 - 2X_2 \leq 4$
$X_1 + 2X_2 \leq 6$
$X_1 + 3X_2 \leq 9$

Non - Negativity

$X_1, X_2, X_3 \geq 0$

24. Solve to maximize $Z = 15X_1 + 3X_2 + 2X_3$

Subject to the constraints:

$4x_1 + 2X_2 + X_3 \geq 20$
$-2X_1 + 4X_3 \geq -12$
$X_1 + X_2 - 2X_3 \leq 30$
$X_1, X_2, X_3 \geq 0$

25. Use simplex method to

maximize $Z = 12X_1 + 2X_2 + X_3$

Subject to the constraints:

$X_1 + x_2 - 2X_3 \leq 20$
$4X_1 + 2X_2 + X_3 \leq 12$
$-2X_1 + 3X_3 \geq -12$
$X_1, X_2, X_3 \geq 0$

26. Solve the following

Maximize $Z = 5X_1 + 3X_2$

Subject to: $X_1 \leq 12$
$X_1 + 2X_2 \geq 14$
$X_2 \leq 5$

$X_1 \geq 4$

$X_1, X_2 \geq 0$

27. Maximize $Z = 10X_1 + X_2 + 2X_3$

subject to the constraints:

$X_1 + x_2 - 2X_3 \leq 10$

$4X_1 = 20$

$-2X_1 + 3X_3 \geq -12$

Non – Negativity

$X_1, X_2, X_3 \geq 0$

28. Solve to maximize $Z = 2X_1 + 3X_2$

Subject to the constraints:

$-X_1 - 2X_2 \leq 4$

$X_1 + 2X_2 \leq 6$

$X_1 + 3X_2 \leq 9$

Non - Negativity

$X_1, X_2, X_3 \geq 0$

29. Solve to maximize $Z = 15X_1 + 3X_2 + 2X_3$

Subject to the constraints:

$4x_1 + 2X_2 + X_3 \geq 20$

$-2X_1 + 4X_3 \geq -12$

$X_1 + X_2 - 2X_3 \leq 30$

$X_1, X_2, X_3 \geq 0$

30. Use simplex method to

maximize $Z = 12X_1 + 2X_2 + X_3$

Subject to the constraints:

$X_1 + x_2 - 2X_3 \leq 20$

$4X_1 + 2X_2 + X_3 \leq 12$

$-2X_1 + 3X_3 \geq -12$

$X_1, X_2, X_3 \geq 0$

31. Solve the following

Maximize $Z = 5X_1 + 3X_2$

Subject to: $X_1 \leq 12$

$X_1 + 2X_2 \geq 14$

$X_2 \leq 5$

$X_1 \geq 4$

$X_1, X_2 \geq 0$

32. Maximize $Z = 10X_1 + X_2 + 2X_3$

subject to the constraints:

$X_1 + x_2 - 2X_3 \leq 10$

$4X_1 = 20$

$-2X_1 + 3X_3 \geq -12$

Non – Negativity

$X_1, X_2, X_3 \geq 0$

33. Solve to maximize $Z = 2X_1 + 3X_2$

Subject to the constraints:

$-X_1 - 2X_2 \leq 4$

$X_1 + 2X_2 \leq 6$

$X_1 + 3X_2 \leq 9$

Non - Negativity

$X_1, X_2, X_3 \geq 0$

34. Solve to maximize $Z = 15X_1 + 3X_2 + 2X_3$

Subject to the constraints:

$4x_1 + 2X_2 + X_3 \geq 20$

$-2X_1 + 4X_3 \geq -12$

$X_1 + X_2 - 2X_3 \leq 30$

$X_1, X_2, X_3 \geq 0$

35. Use simplex method to
maximize $Z = 12X_1 + 2X_2 + X_3$
Subject to the constraints:
$X_1 + x_2 - 2X_3 \leq 20$
$4X_1 + 2X_2 + X_3 \leq 12$
$-2X_1 + 3X_3 \geq -12$
$X_1, X_2, X_3 \geq 0$

36. Solve the following
Maximize $Z = 5X_1 + 3X_2$
Subject to: $X_1 \leq 12$
$X_1 + 2X_2 \geq 14$
$X_2 \leq 5$
$X_1 \geq 4$
$X_1, X_2 \geq 0$

37. Maximize $Z = 10X_1 + X_2 + 2X_3$
subject to the constraints:
$X_1 + x_2 - 2X_3 \leq 10$
$4X_1 = 20$
$-2X_1 + 3X_3 \geq -12$
Non – Negativity
$X_1, X_2, X_3 \geq 0$

38. Solve to maximize $Z = 2X_1 + 3X_2$
Subject to the constraints:
$-X_1 - 2X_2 \leq 4$
$X_1 + 2X_2 \leq 6$
$X_1 + 3X_2 \leq 9$
Non - Negativity
$X_1, X_2, X_3 \geq 0$

39. Solve to maximize $Z = 15X_1 + 3X_2 + 2X_3$
Subject to the constraints:
$4x_1 + 2X_2 + X_3 \geq 20$
$-2X_1 + 4X_3 \geq -12$
$X_1 + X_2 - 2X_3 \leq 30$
$X_1, X_2, X_3 \geq 0$

40. Use simplex method to
maximize $Z = 12X_1 + 2X_2 + X_3$
Subject to the constraints:
$X_1 + x_2 - 2X_3 \leq 20$
$4X_1 + 2X_2 + X_3 \leq 12$
$-2X_1 + 3X_3 \geq -12$
$X_1, X_2, X_3 \geq 0$

Solve the following LPPs graphically

41. Solve the following
Maximize $Z = 5X_1 + 3X_2$
Subject to: $X_1 \leq 12$
$X_1 + 2X_2 \geq 14$
$X_2 \leq 5$
$X_1 \geq 4$
$X_1, X_2 \geq 0$

42. Maximize $Z = 10X_1 + X_2 + 2X_3$
subject to the constraints:
$X_1 + x_2 - 2X_3 \leq 10$
$4X_1 = 20$
$-2X_1 + 3X_3 \geq -12$
Non – Negativity
$X_1, X_2, X_3 \geq 0$

43. Solve to maximize $Z = 2X_1 + 3X_2$

Subject to the constraints:

$-X_1 - 2X_2 \leq 4$

$X_1 + 2X_2 \leq 6$

$X_1 + 3X_2 \leq 9$

Non - Negativity

$X_1, X_2, X_3 \geq 0$

44. Solve the following

Maximize $Z = 5X_1 + 3X_2$

Subject to: $X_1 \leq 12$

$X_1 + 2X_2 \geq 14$

$X_2 \leq 5$

$X_1 \geq 4$

$X_1, X_2 \geq 0$

45. Solve to maximize $Z = 2X_1 + 3X_2$

Subject to the constraints:

$-X_1 - 2X_2 \leq 4$

$X_1 + 2X_2 \leq 6$

$X_1 + 3X_2 \leq 9$

Non - Negativity

$X_1, X_2, X_3 \geq 0$

46. Maximize $Z = 5X_1 + 3X_2$

Subject to: $X_1 \leq 12$

$X_1 + 2X_2 \geq 14$

$X_2 \leq 5$

$X_1 \geq 4$

$X_1 X_2 \geq 0$

47. Maximize $Z = 10X_1 + X_2 + 2X_3$

subject to the constraints:

$X_1 + x_2 - 2X_3 \leq 10$

$4X_1 = 20$

$-2X_1 + 3X_3 \geq -12$

Non – Negativity

$X_1, X_2, X_3 \geq 0$

48. Solve to maximize $Z = 12X_1 + 3X_2$

Subject to the constraints:

$-X_1 - 2X_2 \leq 4$

$X_1 + 2X_2 \leq 16$

$X_1 + 3X_2 \leq 24$

$X_1, X_2, X_3 \geq 0$

49. Maximize $Z = 5X_1 + 3X_2$

Subject to: $X_1 \leq 12$

$X_1 + 2X_2 \geq 14$

$X_2 \leq 5$

$X \geq 4$

$X_1, X_2 \geq 0$

50. Solve to maximize $Z = 2X_1 + 3X_2$

Subject to the constraints:

$-X_1 - 2X_2 \leq 4$

$X_1 + 2X_2 \leq 6$

$X_1 + 3X_2 \leq 9$

$X_1, X_2, X_3 \geq 0$

51. Solve the following

Maximize $Z = 5X_1 + 3X_2$

Subject to: $X_1 \leq 12$

$X_1 + 2X_2 \geq 14$

$X_2 \leq 5$

$X_1 \geq 4$

$X_1, X_2 \geq 0$

52. Solve to maximize $Z = 2X_1 + 3X_2$

Subject to the constraints:

$-X_1 - 2X_2 \leq 4$

$X_1 + 2X_2 \leq 6$

$X_1 + 3X_2 \leq 9$

Non - Negativity

$X_1, X_2, X_3 \geq 0$

53. Solve the following

Maximize $Z = 5X_1 + 3X_2$

Subject to: $X_1 \leq 12$

$X_1 + 2X_2 \geq 14$

$X_2 \leq 5$

$X_1 \geq 4$

$X_1, X_2 \geq 0$

54. Solve to maximize $Z = 2X_1 + 3X_2$

Subject to the constraints:

$-X_1 - 2X_2 \leq 4$

$X_1 + 2X_2 \leq 6$

$X_1 + 3X_2 \leq 9$

Non - Negativity

$X_1, X_2, X_3 \geq 0$

55. Solve the following

Maximize $Z = 5X_1 + 3X_2$

Subject to: $X_1 \leq 12$

$X_1 + 2X_2 \geq 14$

$X_2 \leq 5$

$X_1 \geq 4$

$X_1, X_2 \geq 0$

56. Solve to maximize $Z = 2X_1 + 3X_2$

Subject to the constraints:

$-X_1 - 2X_2 \leq 4$

$X_1 + 2X_2 \leq 6$

$X_1 + 3X_2 \leq 9$

Non - Negativity

$X_1, X_2, X_3 \geq 0$

57. Solve the following

Maximize $Z = 5X_1 + 3X_2$

Subject to: $X_1 \leq 12$

$X_1 + 2X_2 \geq 14$

$X_2 \leq 5$

$X_1 \geq 4$

$X_1, X_2 \geq 0$

58. Solve to maximize $Z = 2X_1 + 3X_2$

Subject to the constraints:

$-X_1 - 2X_2 \leq 4$

$X_1 + 2X_2 \leq 6$

$X_1 + 3X_2 \leq 9$

Non - Negativity

$X_1, X_2, X_3 \geq 0$

59. Solve to maximize $Z = 6X_1 + 9X_2$

Subject to the constraints:

$-X_1 - 2X_2 \leq 8$

$X_1 + 2X_2 \leq 12$

$X_1 + 3X_2 \leq 18$

$X_1, X_2, X_3 \geq 0$

60. Maximize $Z = 20X_1 + 30X_2$

Subject to the constraints:

$5X_1\ 10X_2 \leq 40$

$3X_1 + 2X_2 \leq 60$

$2X_1 + 3X_2 \leq 120$

Non - Negativity

$X_1, X_2, X_3 \geq 0$

61. Solve the following

Minimize $Z = 5X_1 + 3X_2$

Subject to: $X_1 \leq 12$

$X_1 + 2X_2 \geq 14$

$X_2 \leq 5$

$X_1 \geq 4$

$X_1, X_2 \geq 0$

62. Minimize $Z = 10X_1 + X_2 + 2X_3$

subject to the constraints:

$X_1 + x_2 - 2X_3 \leq 10$

$4X_1 = 20$

$-2X_1 + 3X_3 \geq -12$

Non – Negativity

$X_1, X_2, X_3 \geq 0$

63. Solve to minimize $Z = 2X_1 + 3X_2$

Subject to the constraints:

$-X_1 - 2X_2 \leq 4$

$X_1 + 2X_2 \leq 6$

$X_1 + 3X_2 \leq 9$

Non - Negativity

$X_1, X_2, X_3 \geq 0$

64. Solve the following

Minimize $Z = 5X_1 + 3X_2$

Subject to: $X_1 \leq 12$

$X_1 + 2X_2 \geq 14$

$X_2 \leq 5$

$X_1 \geq 4$

$X_1, X_2 \geq 0$

65. Solve to minimize $Z = 2X_1 + 3X_2$

Subject to the constraints:

$-X_1 - 2X_2 \leq 4$

$X_1 + 2X_2 \leq 6$

$X_1 + 3X_2 \leq 9$

Non - Negativity

$X_1, X_2, X_3 \geq 0$

66. Minimize $Z = 5X_1 + 3X_2$

Subject to: $X_1 \leq 12$

$X_1 + 2X_2 \geq 14$

$X_2 \leq 5$

$X_1 \geq 4$

$X_1 X_2 \geq 0$

67. Minimize $Z = 10X_1 + X_2 + 2X_3$

subject to the constraints:

$X_1 + x_2 - 2X_3 \leq 10$

$4X_1 = 20$

$-2X_1 + 3X_3 \geq -12$

Non – Negativity

$X_1, X_2, X_3 \geq 0$

68. Solve to minimize $Z = 2X_1 + 3X_2$

Subject to the constraints:

$-X_1 - 2X_2 \leq 4$

$X_1 + 2X_2 \leq 6$

$X_1 + 3X_2 \leq 9$

$X_1, X_2, X_3 \geq 0$

69. Minimize $Z = 5X_1 + 3X_2$

Subject to: $X_1 \leq 12$

$X_1 + 2X_2 \geq 14$

$X_2 \leq 5$

$X \geq 4$

$X_1, X_2 \geq 0$

70. Solve to minimize $Z = 2X_1 + 3X_2$

Subject to the constraints:

$-X_1 - 2X_2 \leq 4$

$X_1 + 2X_2 \leq 6$

$X_1 + 3X_2 \leq 9$

$X_1, X_2, X_3 \geq 0$

71. Solve the following

Minimize $Z = 5X_1 + 3X_2$

Subject to: $X_1 \leq 12$

$X_1 + 2X_2 \geq 14$

$X_2 \leq 5$

$X_1 \geq 4$

$X_1, X_2 \geq 0$

72. Solve to minimize $Z = 2X_1 + 3X_2$

Subject to the constraints:

$-X_1 - 2X_2 \leq 4$

$X_1 + 2X_2 \leq 6$

$X_1 + 3X_2 \leq 9$

Non - Negativity

$X_1, X_2, X_3 \geq 0$

73. Solve the following

Minimize $Z = 5X_1 + 3X_2$

Subject to: $X_1 \leq 12$

$X_1 + 2X_2 \geq 14$

$X_2 \leq 5$

$X_1 \geq 4$

$X_1, X_2 \geq 0$

74. Solve to minimize $Z = 2X_1 + 3X_2$

Subject to the constraints:

$-X_1 - 2X_2 \leq 4$

$X_1 + 2X_2 \leq 6$

$X_1 + 3X_2 \leq 9$

Non - Negativity

$X_1, X_2, X_3 \geq 0$

75. Solve the following

Minimize $Z = 5X_1 + 3X_2$

Subject to: $X_1 \leq 12$

$X_1 + 2X_2 \geq 14$

$X_2 \leq 5$

$X_1 \geq 4$

$X_1, X_2 \geq 0$

76. Solve to minimize $Z = 2X_1 + 3X_2$

Subject to the constraints:

$-X_1 - 2X_2 \leq 4$

$X_1 + 2X_2 \leq 6$

$X_1 + 3X_2 \leq 9$

Non - Negativity

$X_1, X_2, X_3 \geq 0$

77. Solve the following

Minimize $Z = 5X_1 + 3X_2$

Subject to: $X_1 \leq 12$

$X_1 + 2X_2 \geq 14$

$X_2 \leq 5$

$X_1 \geq 4$

$X_1, X_2 \geq 0$

78. Solve to minimize $Z = 2X_1 + 3X_2$

Subject to the constraints:

$-X_1 - 2X_2 \leq 4$

$X_1 + 2X_2 \leq 6$

$X_1 + 3X_2 \leq 9$

Non - Negativity

$X_1, X_2, X_3 \geq 0$

79. Solve to minimize $Z = 6X_1 + 9X_2$

Subject to the constraints:

$-X_1 - 2X_2 \leq 8$

$X_1 + 2X_2 \leq 12$

$X_1 + 3X_2 \leq 18$

$X_1, X_2, X_3 \geq 0$

80. Minimize $Z = 20X_1 + 30X_2$

Subject to the constraints:

$5X_1\ 10X_2 \leq 40$

$3X_1 + 2X_2 \leq 60$

$2X_1 + 3X_2 \leq 120$

Non - Negativity

$X_1, X_2, X_3 \geq 0$

Unit – III
ASSIGNMENT

13. What is an Assignment Model?

Consider the problem of assigning n job to n persons such that job can be assigned to only one person and each person can be allotted only one job.

Mathematical formulation of an assignment problem;

1 Xij=1,j=1,2,…..n

2 Since each job can be done by only person

=1,i=1,2,…n

Therefore , the mathematical model fro an assignment problem is minimize the total cost.

Z= Cij Xij subject to

1. Xij=1 j=1,2,…n
2. Xij=1 i=1,2,…n

14. Explain the Hungarian method for solving an Assignment model.

HUNGARIAN METHOD FOR SOLUTION OF THE ASSIGNMENT PROBLEMS;

The Hungarian method suggested by Mr. KONING of Hungarian as the reduced or the flood's Technique is used for solving assignment problem since it is quite efficient and results in substantial time saving over the other techniques. It involves a rapid reduction of the original matrix and finding a set of n independent zeros, one in each row and column, which results in n optimal solution. The Hungarian method consists of the following steps.

1. Prepare square matrix This step will not be required for n*m (m ≠n) problems, a dummy column or a dummy row, as the case may be added to make the matrix square.

2. Reduce the matrix

Subtract the smallest element of each row from all the elements of the row. So there will be at least one zero in each row. Examine if there is at least one zero in each column. If not, subtract the minimum element of the columns(s) not containing zero from all the elements of that columns. This reduces the elements of the matrix until zero's called zero opportunity costs are obtained in each column.

3) Check whether an optimal assignment can be made in the reduced matrix as not.

For this;

a) Examine rows successively until a row is exactly one unmarked zero is obtained.

- Make an assignment to this single zero by making square around it.
- Cross (X) all other zero's in the same column as they will not be considered for making any more assignment in that column.
- Proceed in this way until all rows have been examined.

b) Now examined column successfully until a column with exactly one unmarked zero is formed.

- Make an assignment there by a making around it and cross(x) any other zero's in the same row.

In case there is no row or column containing single unmarked zero (can be more than one) mark around any unmarked zero arbitrarily and cross (x) all other zero's on its row and column. Proceed in this manner will there is no unmarked zero left in the cost matrix. Repeat sub steps (a) & (b) till one of the following occur.

i) There is one assignment in each row & in each column. In this case the optimal assignment can be made in the current solution i.e., the current feasible solution is a optimal solution.
ii) The minimum no. of crossing all zero's is n, the order of the matrix.
There is same row/column without assignment. In this case optimal assignment cannot be made in the current solution.

4) Find the minimum numbers of lines crossing all zeroes

This consists of following sub steps:

1. Mark the rows do not have assignments.
2. Mark the columns that have zeros in marked rows.
3. Mark the rows that have assignment in marked columns
4. Repeat sub-steps (ii) & (iii) till no more rows and marked columns.
5. Draw straight lines through all unmarked rows and marked columns. This gives the minimum number of lines crossing all zero's, it this number is equal to the order of the matrix, then it is optimal solution, otherwise go to step 5.

5) ITERATE towards optimal solution;

* Examine the uncovered element. Select the smallest element and subtract it from all the uncovered elements.

* Add this smallest element to every element that lies at the intersection of two lines.

* Leave the remaining elements of the matrix as such this yields second basic feasible solution.

6) Repeat step 3 through 5 successively until the number of .line crossing all the zeros becomes equal to the order of the matrix. In such a case every row & column will have one assignment. This indicates that an optimal solution has been obtained.

What are the variations or special cases in the Assignment Problem:

Non square matrix (unbalanced assignment problem) such a problem is found to exist when the number of facilities is not equal to the number of job. Since the Hungarian method of a square matrix fictitious or job may be added and zero cost be assigned the corresponding cell of the matrix. These cells are then treated the same way as the real cost cell during the solution produced.

Maximization problem. Sometimes the assignment problem may deal with maximization of the objective function. The maximization problem has to be changed to minimization before the Hungarian method may be applied, the transformation may be done in either of the following two ways.

a) By subtracting all the elements from the largest element of the matrix
b) By multiplying the matrix elements by -1.

The Hungarian method can then be applied to this equivalent minimization problem to be obtained.

Restriction on assignment: Sometimes technical, space, legal or other restriction do not permit the assignment of a particular job. Such problem can be solved by assignment a very heavy cost to the correspondent cell. Such a job will be automatically excluded form further consideration.

Alteration optional solution: Sometimes it is possible to have two or more ways to strike off all zero elements in the reduced matrix for the given problem. In such case, there will be alternative optional solution with the management since it can select the one which suitable to its requirement.

THE TRAVELLING SALESMAN PROBLEM:

There are a number of cities a salesman visit. The distance between every pair f cities is known. HE starts from his home city passes through each cities once only and return to his home city. The problem is to routes shortest in distance. If the distance be asymmetrical. For example, it takes more time to climb up the hill but less time to come down from the hill, a flight will take more time against the wind direction compared to that in the direction of the wind.

If the salesman is to visit only two cities there is of course, no chaise. If the number of cities id 3(A,B&C) of which starting base id A, there two possible routed A-B-C-D,-C-A-C-B,-DA-A-C-D-B-A-D-B-C,A-D-C-B.

For 11 cities there are more than 31/2 million possible routes and for 21 cities, there are 20!=2.42329*10^8 different routs every a fast electronics computer testing a route every macro second and working 8hrs a day. Would take around 2,31,440 years to find the best solution the salesman has to visit all the n cities, the shortest route will be independent odd the selection of the starting city.

Obviously, the problem is to find the best route without trying each one. Unfortunately, there is no analytical method which can be used satisfactory. However, few computational techniques for solving the problem have been suggested.

Such types of problem arise in the following areas of the management.

i. Postal deliveries.
ii. Inspection
iii. School bus routing
iv. Television relays
v. Assembly lines(etc)

Assignment Problems:

1. Solve the following assignment problem to find the optimal solution. The values given are profit in Rupees.

Jobs	Workers			
	W1	W2	W3	W4
Job 1	2	-2	1	1
Job 2	1	0	3	4
Job 3	2	3	8	6
Job 4	20	20	45	15

2. Solve the following assignment problem to find the optimal solution.

Jobs	**Workers**				
	W1	**W2**	**W3**	**W4**	**W5**
J1	10	13	11	15	6
J2	12	18	21	23	15
J3	8	6	23	18	4
J4	9	4	12	15	20

3. Solve the following transportation problem to find the optimal solution.

Workers	**Jobs**			
	J1	**J2**	**J3**	**J4**
W1	12	10	15	19
W2	12	24	15	32
W3	12	15	16	14

4. Solve the following assignment problem to find the optimal solution.

Jobs	**Workers**				
	W1	**W2**	**W3**	**W4**	**W5**
J1	12	10	15	19	24
J2	12	24	15	32	10
J3	12	15	16	14	13
J4	12	14	18	24	29

5. Solve the following assignment problem to find the optimal solution. The values given are profit in Rupees.

Jobs	Workers			
	W1	W2	W3	W4
Job 1	1	-4	2	1
Job 2	3	0	5	6
Job 3	8	3	8	8
Job 4	45	20	9	10

6. Solve the following assignment problem to find the optimal solution.

Jobs	Workers				
	W1	W2	W3	W4	W5
J1	10	13	11	15	6
J2	12	18	21	23	15
J3	8	6	23	18	4
J4	9	4	12	15	20
J5	21	27	8	8	9

7. Solve the following transportation problem to find the optimal solution.

Workers	Jobs			
	J1	J2	J3	J4
W1	22	12	18	24
W2	12	22	32	22
W3	18	16	18	19

8. Solve the following assignment problem to find the optimal solution.

Jobs	Workers				
	W1	W2	W3	W4	W5
J1	12	15	15	19	12
J2	12	14	15	32	13
J3	12	13	16	14	14
J4	12	19	18	24	15
J5	12	12	19	24	16

9. Solve the following assignment problem.

Jobs	Workers				
	W1	W2	W3	W4	W5
J1	12	10	15	19	24
J2	12	24	15	32	10
J3	12	15	16	14	13
J4	12	14	18	24	29

10. Find the optimal solution for the following assignment problem.

Jobs	Workers				
	W1	W2	W3	W4	W5
J1	12	10	15	19	24
J2	12	24	15	32	10
J3	12	15	16	14	13
J4	12	14	18	24	29
J5	11	13	17	13	12
J6	10	15	9	8	10

11. Solve the following assignment problem to find the optimal solution.

Jobs	Workers				
	W1	W2	W3	W4	W5
J1	10	13	11	15	6
J2	12	18	21	23	15
J3	8	6	23	18	4
J4	9	4	12	15	20
J5	21	27	8	8	9

12. Solve the following assignment problem to find the optimal solution.

Jobs	Workers				
	W1	W2	W3	W4	W5
J1	12	10	15	19	24
J2	12	24	15	32	10
J3	12	15	16	14	13
J4	12	14	18	24	29
J5	12	17	19	24	26

13. Solve the following transportation problem to find the optimal solution.

Jobs	Workers			
	W1	W2	W3	W4
J1	12	10	15	19
J2	12	24	15	32
J3	12	15	16	14
J4	12	14	18	24

14. Solve the following assignment problem to find the optimal solution.

Jobs	Workers			
	W1	W2	W3	W4
J1	12	10	15	19
J2	12	24	15	32
J3	12	15	16	14

J4	12	14	18	24

15. Find the optimal solution from the following transportation problem.

Location	**Warehouse**			
	W1	**W2**	**W3**	**W4**
L1	5	8	2	10
L2	13	12	4	13
L3	4	10	5	15
L4	45	55	25	35

16. Solve the following problem. Values given are profit in Rs.

Location	**Warehouse**			
	W1	**W2**	**W3**	**W4**
L1	2	3	1	-2
L2	2	5	8	3
L3	1	2	3	0
L4	2	3	5	3

17. Find the optimal solution from the following Assignment problem.

Location	**Warehouse**			
	W1	**W2**	**W3**	**W4**
L1	21	12	13	31
L2	11	22	21	10
L3	18	15	5	3
L4	20	20	20	20
L5	18	19	24	16

18. Solve the following assignment problem to find the optimal solution.

Jobs	**Workers**				
	W1	**W2**	**W3**	**W4**	**W5**
J1	12	15	15	19	12
J2	12	14	15	32	13
J3	12	13	16	14	14
J4	12	19	18	24	15
J5	12	12	19	24	16

19. Solve the following transportation problem to find the optimal solution. The values given are profit in Rupees.

Location	**Warehouse**			
	W1	**W2**	**W3**	**W4**
L1	12	3	1	-2
L2	10	2	3	0
L3	2	5	8	3

20. Solve the following assignment problem to find the optimal solution.

Jobs	**Workers**				
	W1	**W2**	**W3**	**W4**	**W5**
J1	12	10	15	19	24
J2	12	24	15	32	10
J3	12	15	16	14	13
J4	12	14	18	24	29
J5	12	17	19	24	26
J6	12	16	18	26	28

21. Four jobs can be processed on four different machines using four operators, each machine doing only one job. The processing time of the machines is given below, find out the best combination so as to minimize the idle time.

Jobs	**Machines**			
	A	**B**	**C**	**D**
I	42	35	28	21
II	30	25	20	15
III	30	25	20	15
IV	24	20	16	12

22. Solve the following assignment problem to find the optimal solution.

Jobs	**Workers**				
	W1	**W2**	**W3**	**W4**	**W5**
J1	12	10	15	19	24

J2	12	24	15	32	10
J3	12	15	16	14	13
J4	12	14	18	24	29
J5	12	17	19	24	26

23. A company faced with the problem of assigning 6 different Machines and 5 different jobs. The cost estimate is as follows in rupees. Solve their problem

Machines	**Jobs**				
	J1	**J2**	**J3**	**J4**	**J5**
1	2.5	5	1	6	1
2	2	5	1.5	7	3
3	3	6.5	2	8	3
4	3.5	6.5	2	9	4.5
5	3.5	7	3	9	6
6	6	9	5	10	6

24. Solve the following assignment problem to find the optimal solution. The values given are profit in Rupees.

Jobs	Workers			
	W1	W2	W3	W4
Job 1	1	-4	2	1
Job 2	3	0	5	6
Job 3	8	3	8	8
Job 4	45	20	9	10

25. Solve the following transportation problem to find the optimal solution.

Workers	**Jobs**			
	J1	**J2**	**J3**	**J4**
W1	12	10	15	19
W2	12	24	15	32
W3	12	15	16	14

26. Solve the following problem. Values given are profit in Rs.

Location	**Warehouse**			
	W1	**W2**	**W3**	**W4**
L1	2	3	1	-2
L2	2	5	8	3
L3	1	2	3	0
L4	2	3	5	3

27. Solve the following assignment problem. The values given are profit in Rupees.

Jobs	Workers			
	W1	W2	W3	W4
Job 1	2	-2	1	1
Job 2	1	0	3	4
Job 3	2	3	8	6
Job 4	20	20	45	15

28. Solve the following problem. Values given are profit in Rs.

Location	**Warehouse**			
	W1	**W2**	**W3**	**W4**
L1	2	3	1	-2
L2	2	8	8	5
L3	1	3	2	0
L4	3	2	5	3

29. Solve the following transportation problem to find the optimal solution.

Workers	**Jobs**			
	J1	**J2**	**J3**	**J4**
W1	46	56	66	76
W2	56	66	76	86
W3	45	54	64	56
W4	54	45	56	65
W5	66	57	48	75

30. Solve the following transportation problem to find the optimal solution.

Workers	Jobs			
	J1	J2	J3	J4
W1	9	8	4	9
W2	6	5	8	3
W3	9	8	8	4
W4	0	1	7	8
W5	6	-4	3	8

31. Solve the following assignment problem to find the optimal solution.

Jobs	Workers				
	W1	W2	W3	W4	W5
J1	10	13	11	15	6
J2	12	18	21	23	15
J3	8	6	23	18	4
J4	9	4	12	15	20
J5	21	27	8	8	9

32. Solve the following assignment problem to find the optimal solution. The values given are profit in Rupees.

Jobs	Workers			
	W1	W2	W3	W4
Job 1	2	-2	1	1
Job 2	1	0	3	4
Job 3	2	3	8	6
Job 4	20	20	45	15

33. Solve the following assignment problem to find the optimal solution.

Jobs	Workers				
	W1	W2	W3	W4	W5
J1	10	13	11	15	6
J2	12	18	21	23	15
J3	8	6	23	18	4
J4	9	4	12	15	20

34. A company has 5 machines and 5 jobs to be done. The return in Rs. Of assigning the i^{th} machines to the j^{th} job.
i,j = 1,2,3,4,5 is as follows:

Assign the 5 jobs to the 5 machines so as to maximum the total profit.

Machines		Jobs			
	1	2	3	4	5
A	5	11	10	12	4
B	2	4	6	3	5
C	3	12	5	14	6
D	6	14	4	11	7
E	7	9	8	12	5

35. Solve the following assignment problem to find the optimal solution.

Jobs	Workers				
	W1	W2	W3	W4	W5
J1	12	10	15	19	24
J2	12	24	15	32	10
J3	12	15	16	14	13
J4	12	14	18	24	29
J5	12	17	19	24	26

36. Find the optimal solution.

Jobs	Workers				
	W1	W2	W3	W4	W5
J1	12	15	15	19	12
J2	12	14	15	32	13
J3	12	13	16	14	14
J4	12	19	18	24	15
J5	12	12	19	24	16

37. Solve the following assignment problem. The values given are profit in Rupees.

Jobs	Workers

	W1	W2	W3	W4
Job 1	2	-2	1	1
Job 2	1	0	3	4
Job 3	2	3	8	6
Job 4	20	20	45	15

38. Solve the following assignment problem to find the optimal solution.

Jobs	Workers				
	W1	W2	W3	W4	W5
J1	12	10	15	19	24
J2	12	24	15	32	10
J3	12	15	16	14	13
J4	12	14	18	24	29

39. Solve the following assignment problem.

Jobs	Workers				
	W1	W2	W3	W4	W5
J1	12	10	15	19	24
J2	12	24	15	32	10
J3	12	15	16	14	13
J4	12	14	18	24	29

40. Solve the following assignment problem to find the optimal solution.

Jobs	Workers			
	W1	W2	W3	W4
J1	12	8	12	16
J2	13	22	15	32
J3	12	14	18	13
J4	13	15	17	22

41. Find the optimal solution for the following assignment problem.

Jobs	Workers				
	W1	W2	W3	W4	W5
J1	12	10	15	19	24
J2	12	24	15	32	10
J3	12	15	16	14	13
J4	12	14	18	24	29
J5	11	13	17	13	12
J6	10	15	9	8	10

42. Solve the following assignment problem to find the optimal solution.

Jobs	Workers				
	W1	W2	W3	W4	W5
J1	10	13	11	15	6
J2	12	18	21	23	15
J3	8	6	23	18	4
J4	9	4	12	15	20
J5	21	27	8	8	9

43. Solve the following assignment problem to find the optimal solution.

Jobs	Workers				
	W1	W2	W3	W4	W5
J1	12	10	15	19	24
J2	12	24	15	32	10
J3	12	15	16	14	13
J4	12	14	18	24	29
J5	12	17	19	24	26

44. A company faced with the problem of assigning 6 different Machines and 5 different jobs. The cost estimate is as follows in rupees. Solve their problem

Machines	Jobs				
	J1	J2	J3	J4	J5
1	2.5	5	1	6	1
2	2.25	5	1.5	7.5	3
3	3	6.5	2	8	3
4	3.5	6.5	2	9.25	4.5
5	3.5	7	3	9	6
6	6	9	5	10.5	6

45. Solve the following assignment problem.

Jobs	Workers				
	W1	W2	W3	W4	W5
J1	12	15	15	19	12

J2	12	14	15	32	13
J3	12	13	16	14	14
J4	12	19	18	24	15
J5	12	12	19	24	16

46. Solve the following transportation problem to find the optimal solution. The values given are profit in Rupees.

Location	**Warehouse**			
	W1	**W2**	**W3**	**W4**
L1	12	3	1	-2
L2	10	2	3	0
L3	2	5	8	3

47. Solve the following assignment problem to find the optimal solution.

Jobs	**Workers**			
	W1	**W2**	**W3**	**W4**
J1	12	10	15	19
J2	12	24	15	32
J3	12	15	16	14
J4	12	14	18	24

48. Solve the following transportation problem to find the optimal solution. The values given are profit in Rupees. Values given in the cells are transportation costs in Rupees.

Location	**Warehouse**			
	W1	**W2**	**W3**	**W4**
L1	12	3	1	-2
L2	10	2	3	0
L3	2	5	8	3
L4	5	7	8	12

49. Four jobs can be processed on four different machines using four operators, each machine doing only one job. The processing time of the machines is given below, find out the best combination so as to minimize the idle time.

Jobs	**Machines**			
	A	**B**	**C**	**D**
I	42	35	28	21
II	30	25	20	15
III	30	25	20	15
IV	24	20	16	12

50. Solve the following problem. Values given are profit in Rs.

Location	**Warehouse**			
	W1	**W2**	**W3**	**W4**
L1	2	3	1	-2
L2	2	5	8	3
L3	1	2	3	0
L4	2	3	5	3

51. Solve the following transportation problem to find the optimal solution.

Workers	**Jobs**			
	J1	**J2**	**J3**	**J4**
W1	12	10	15	19
W2	12	24	15	32
W3	12	15	16	14

52. Find the optimal solution from the following Assignment problem.

Location	**Warehouse**			
	W1	**W2**	**W3**	**W4**
L1	31	13	12	21
L2	12	22	33	18
L3	21	15	5	3
L4	20	20	20	20
L5	18	19	24	16

53. A company has 5 machines and 5 jobs to be done. The return in Rs. Of assigning the i^{th} machines to the j^{th} job.

i,j = 1,2,3,4,5 is as follows:

Assign the 5 jobs to the 5 machines so as to maximum the total profit.

Machines	Jobs				
	1	2	3	4	5
A	6	9	7	8	10
B	4	5	5	6	5
C	3	7	10	6	15
D	6	12	3	11	7
E	7	9	8	12	5

54. Find the optimal solution from the following transportation problem.

Location	Warehouse			
	W1	W2	W3	W4
L1	5	8	2	10
L2	13	12	4	13
L3	4	10	5	15
L4	45	55	25	35

55. Solve the following problem. Values given are profit in Rs.

Location	Warehouse			
	W1	W2	W3	W4
L1	2	3	1	-2
L2	2	5	8	3
L3	1	2	3	0
L4	2	3	5	3

56. A solicitors firm employs typists on hourly piece rate basis for their daily work. There are five typists for services and their charges and speeds are different according to an earlier understanding only one hob is given to one typist and the typist is paid for full hour if he works for a fraction of an hour. Find the least cost allocation for the following data:

Typist	Rate Per Hour	No. of Pagers typed per	Job	No. of Pages
		hour		
A	5	12	P	199
B	6	14	Q	175
C	3	8	R	145
D	4	10	S	298
E	4	11	T	178

57. Solve the following assignment problem to find the optimal solution. The values given are profit in Rupees.

Jobs	Workers			
	W1	W2	W3	W4
Job 1	12	-2	11	10
Job 2	13	0	15	9
Job 3	12	6	9	18
Job 4	20	20	45	15

58. A company has taken the third floor of a building for rent with a view to locate one of their zonal offices. There are five main rooms in this floor to be assigned to five managers. Each room has its own advantages and disadvantages. Each of the managers was asked to rank the rooms in order of their preferences. Their preferences were recorded. The rooms are numbered as 301, 302, 303, 304 and 305

	MANAGER				
Ranks	M1	M2	M3	M4	M5
1	301	303	303	302	302
2	303	304	304	305	301
3	304	305	301	303	304
4		301	305	304	
5			302		

Note: Blank space means the manager does not prefer that particular room.

59. Find the optimal solution for the following traveling salesman's problem

From	To				
	A	B	C	D	E
A	-	4	7	3	4
B	4	-	6	3	4
C	7	6	-	7	5
D	3	3	3	-	7
E	4	4	5	7	-

60. A small garment making unit has five tailors stitching five different types of garments. All the five tailors are capable of stitching all the types of garments. The output per tailor and the profit in rupees for each type of garments are given below:

Tailors	Garments				
	1	**2**	**3**	**4**	**5**
A	7	9	4	8	6
B	4	8	5	7	9
C	8	5	7	10	8
D	5	5	8	9	10
E	7	8	10	9	9
Profit per garment (Rs.)	2	4	3	2	4

61. Find the optimal solution for the following traveling salesman's problem

From	**To**			
	Delhi	**Chandigarh**	**Shimla**	**Jaipur**
Delhi	-	8	3	12
Chandigarh	4	-	9	6
Shimla	5	4	-	8
Jaipur	6	3	7	-

62. Find the optimal solution for the following traveling salesman's problem

From	**To**			
	Delhi	**Chandigarh**	**Shimla**	**Jaipur**
Delhi	-	10	5	10
Chandigarh	6	-	7	8
Shimla	4	5	-	6
Jaipur	5	2	6	-

63. XYZ Airlines operating 7 days a week has given the following time table. Crews must have a minimum layover of 5 hours between flights. Obtain the pairing of flights so that the layover can be kept at the minimum. Also find the base of each crew.

Chennai – Mumbai				Mumbai - Chennai		
Flight No.	Dep	Arr		Flight No.	Dep	Arr
A1	6 am	8 am		B1	8 am	10 am
A2	8 am	10 am		B2	9 am	11 am
A3	2 pm	4 pm		B3	7 pm	9 pm

64. XYZ Airlines operating 7 days a week has given the following time table. Crews must have a minimum layover of 5 hours between flights. Obtain the pairing of flights so that the layover can be kept at the minimum. Also find the base of each crew.

Delhi – Mumbai				Delhi - Chennai		
Flight No.	Dep	Arr		Flight No.	Dep	Arr
A1	6 am	7 am		B1	7 am	8 am
A2	8 am	9 am		B2	11 am	12 noon
A3	2 pm	3 pm		B3	8 pm	9 pm

65. Find the optimal solution for the following traveling salesman's problem

From	**To**			
	Delhi	**Agra**	**Shimla**	**Jaipur**
Delhi	-	2	3	5
Agra	6	-	8	10
Shimla	10	9	-	3

Jaipur	5	2	6	-

66. Find the optimal solution for the following traveling salesman's problem

From	To				
	A	B	C	D	E
A	-	3	7	5	4
B	5	-	8	3	4
C	6	5	-	7	3
D	2	4	3	-	8
E	4	6	5	7	-

67. Solve the traveling salesman problem (The figures in the cells are costs)

From	**To**			
	Delhi	**Chandigarh**	**Shimla**	**Jaipur**
Delhi	-	10	5	10
Chandigarh	6	-	7	8
Shimla	4	5	-	6
Jaipur	5	2	6	-

68. A solicitors firm employs typists on hourly piece rate basis for their daily work. There are five typists for services and their charges and speeds are different according to an earlier understanding only one hob is given to one typist and the typist is paid for full hour if he works for a fraction of an hour. Find the least cost allocation for the following data:

Typist	Rate Per Hour	No. of Pagers typed per hour	Job	No. of Pages
A	5	11	P	189
B	6	15	Q	165
C	3	9	R	155
D	4	11	S	300
E	4	12	T	188

69. There are five routine works to be performed and there are five clerks to do them. Each one is capable of doing any of the jobs, but one will do only one job on any one day. The following table gives the efficiencies in performing the jobs by each of the clerks.

Clerks		**Jobs**			
	A	**B**	**C**	**D**	**E**
1	70	65	80	72	69
2	62	-	70	75	74
3	-	74	68	72	73
4	-	-	78	71	72
5	71	73	75	-	-

70. An organization producing 4 different products viz., A,B,C and D having 4 operators viz., P,Q,R and S who are capable of producing any of the four products, works effectively 7 hours a day. The times in minutes required for each operator for producing each of the products are given below in the nine cells of the following matrix with profits in Rs per unit.

Operator	**Product**			
	A	**B**	**C**	**D**
P	6	10	14	12
Q	7	5	3	4
R	6	7	10	10
S	20	12	15	15
Profit (Rs)	3	2	4	1

Find out the assignment of operators which will maximize the profit

71. Azeem Corporation has four plants each of which can manufacture any one of four products. Production costs differ from one plant to another as do sales revenue. Given the revenue and cost data below, obtain which product each plant should produce to maximize profit:

Revenue in Rupees

Plant	**Product (Rs.)**

	1	2	3	4
A	50	68	49	62
B	60	70	51	74
C	55	67	53	70
D	58	65	54	69

Production Cost in Rupees

Plant	Product (Rs.)			
	1	2	3	4
A	49	60	45	61
B	55	63	45	69
C	52	62	49	58
D	55	64	48	66

72. Solve the following transportation problem to find the optimal solution.

Workers	Jobs			
	J1	J2	J3	J4
W1	9	8	4	9
W2	6	5	8	3
W3	9	8	8	4
W4	0	1	7	8
W5	6	-4	3	8

73. Azeem Corporation has four plants each of which can manufacture any one of four products. Production costs differ from one plant to another as do sales revenue. Given the revenue and cost data below, obtain which product each plant should produce to maximize profit:

Revenue in Rupees

Plant	Product (Rs.)			
	1	2	3	4
A	55	65	59	63
B	59	71	55	71
C	54	57	53	70
D	58	65	59	69

Production Cost in Rupees

Plant	Product (Rs.)

	1	2	3	4
A	49	60	45	61
B	55	63	45	69
C	52	62	49	58
D	55	64	48	66

74. The captain of a cricket team has to allot five batting positions to five batsmen. The average runs score by each batsman at these positions are as follows:

Batsman	Batting Positions				
	I	II	III	IV	V
P	40	40	35	25	50
Q	42	30	16	25	27
R	50	48	40	60	50
S	20	19	20	18	25
T	58	60	59	55	53

a. Find the assignment of batsmen to positions which would give the maximum number of runs.

b. If another batsmen 'U' with the following average runs in batting positions as given below:

Batting Positions	I	II	III	IV	V
Average Runs	45	52	38	50	49

is added to the team, should he be included to play in the team? If so who will be replaced by him?

75. Average time taken by an operator on a specific machine is tabulated below. The management is considering replacing one of the old machines by a new machine and the estimated time for operation by each operator of the new machine is also indicated.

Operator	Machines						
	1	2	3	4	5	6	New
A	10	12	8	10	8	10	11
B	9	10	8	7	8	9	10

C	8	7	8	8	8	6	8
D	12	13	14	14	15	14	11
E	9	9	9	8	8	10	9
F	7	8	9	9	9	8	8

a. Find out an allocation of operators to the old machine to achieve a minimum operation time.
b. Reset the problem with the new machine and find out the allocation of the operators to each machine and comment whether is advantageous to replace an old machine to achieve a deduction in operation time only.
c. However will the operators be re-allotted to the machines after replacement?

76. Five swimmers are eligible to compete in a relay team which is to consist of four swimmers swimming four different swimming styles: Back stroke, Breast stroke, free style and butterfly. The time taken for the five swimmers – Anand, Baskar, Chandra, Darwin and Evan to cover a distance of 100 metres in various swimming styles are given below in **minutes:seconds**. Anand swims the back stroke in 1:09, the breast stroke in 1:05 and has never competed in the free style or butterfly. Baskar is a free style specialist averaging 1:01 for the 100 metres but can also swim the breast stroke in 1:16 and butterfly in 1:20. Chandra swims all styles back stroke 1:10, butterfly 1:12, free style 1:05 and breast stroke 1:20. Darwin swims only the butterfly 1:11. Evan swims the back stroke 1:20, the breast stroke 1:16, free style 1:06 and the butterfly 1:10. Which swimmer should be assigned to which swimming style? Who will not be in the relay?

77. To stimulate interest and provide an atmosphere for intellectual discussion a finance faculty in a management school, decides to hold special seminars on four contemporary topics-leasing, portfolio management, private mutual funds, swaps and options. Such seminars should be held once per week in the afternoons. However, scheduling these seminars (one for each topic and not more than one seminar per afternoon) has to be done carefully so that the number of students unable to attend is kept to a minimum. A careful study indicates that the number of students who cannot attend a particular seminar on a specific day is as follows:

Days	Leasing	Portfolio Management	Private Mutual Fund	Swaps and Options
Mon	50	40	60	20
Tue	40	40	40	30
Wed	60	20	30	20
Thur	30	30	20	30
Fri	10	20	10	30

Find out the optimum schedule of the seminars. Find out the total number of students who will be missing at least one seminar.

78. The secretary of a school is taking bids on the city's four school bus routes. Four companies have made the bids as detailed in the following table:

Company	Bids in Rupees			
	Route 1	Route 2	Route 3	Route 4
C1	4000	5000	-	-
C2	-	4000	-	4000
C3	3000	-	2000	-
C4	-	-	4000	5000

Suppose each bidder can be assigned only one route. Use the assignment model to minimize the school's cost of running the four bus routes.

79. XYZ Airlines operating 7 days a week has given the following time table. Crews must have a minimum layover of 3 hours between flights. Obtain the pairing of flights so that the layover can be kept at the minimum. Also find the base of each crew.

Chennai – Mumbai		Mumbai - Chennai

Flight No.	Dep	Arr		Flight No.	Dep	Arr
A1	6.15 am	7.45 am		B1	8 am	9.30 am
A2	8.30 am	10.15 am		B2	9.15 am	11 am
A3	2 pm	4 pm		B3	7 pm	9.15 pm

80. Big Corporation has four plants each of which can manufacture any one of four products. Production costs differ from one plant to another as do sales revenue. Given the revenue and cost data below, obtain which product each plant should produce to maximize profit:

Revenue in Rupees

Plant	Product (Rs.)			
	1	2	3	4
A	60	65	59	63
B	59	71	55	71
C	54	57	53	70
D	58	65	59	69

Production Cost in Rupees

Plant	Product (Rs.)			
	1	2	3	4
A	59	61	44	45
B	54	55	63	43
C	69	52	58	55
D	58	48	55	60

81. Big Corporation has four plants each of which can manufacture any one of four products. Production costs differ from one plant to another as do sales revenue. Given the revenue and cost data below, obtain which product each plant should produce to maximize profit:

Revenue in Rupees

Plant	Product (Rs.)			
	1	2	3	4
A	60	65	59	63
B	59	71	55	71
C	54	57	53	70
D	58	65	59	69

Production Cost in Rupees

Plant	Product (Rs.)			
	1	2	3	4
A	59	61	44	45
B	54	55	63	43
C	69	52	58	55
D	58	48	55	60

82. An organization producing 4 different products viz., A,B,C and D having 4 operators viz., P,Q,R and S who are capable of producing any of the four products, works effectively 7 hours a day. The times in minutes required for each operator for producing each of the products are given below in the nine cells of the following matrix with profits in Rs per unit.

Operator	Product			
	A	B	C	D
P	6	10	14	12
Q	8	6	4	6
R	7	9	11	10
S	20	12	15	15
Profit (Rs)	1	4	3	2

Find out the assignment of operators which will maximize the profit.

83. A solicitors firm employs typists on hourly piece rate basis for their daily work. There are five typists for services and their charges and speeds are different according to an earlier understanding only one hob is given to one typist and the typist is paid for full hour if he works for a fraction of an hour. Find the least cost allocation for the following data:

Typist	Rate Per Hour	No. of Pagers typed per hour	Job	No. of Pages
A	5	12	P	199
B	6	14	Q	175
C	3	8	R	145
D	4	10	S	298

E	4	11	T	178

84. XYZ Airlines operating 7 days a week has given the following time table. Crews must have a minimum layover of 4 hours between flights. Obtain the pairing of flights so that the layover can be kept at the minimum. Also find the base of each crew.

Chennai – Mumbai				Mumbai - Chennai		
Flight No.	Dep	Arr		Flight No.	Dep	Arr
A1	6 am	8 am		B1	8 am	10 am
A2	8 am	10 am		B2	9 am	11 am
A3	2 pm	4 pm		B3	7 pm	9 pm

85. Find the optimal solution for the following traveling salesman's problem

From	To				
	A	B	C	D	E
A	-	4	7	3	4
B	4	-	6	3	4
C	7	6	-	7	5
D	3	3	3	-	7
E	4	4	5	7	-

86. XYZ Airlines operating 7 days a week has given the following time table. Crews must have a minimum layover of 2 hours between flights. Obtain the pairing of flights so that the layover can be kept at the minimum. Also find the base of each crew.

Delhi – Mumbai				Delhi - Chennai		
Flight No.	Dep	Arr		Flight No.	Dep	Arr
A1	6 am	7 am		B1	7 am	8 am
A2	8 am	9.15 am		B2	10 am	11 noon
A3	2.15 pm	3.30 pm		B3	8 pm	9 pm

87. Five swimmers are eligible to compete in a relay team which is to consist of four swimmers swimming four different swimming styles: Back stroke, Breast stroke, free style and butterfly. The time taken for the five swimmers – Amar, Akbar, Antony, Ashoka and Alex to cover a distance of 100 metres in various swimming styles are given below in **minutes:seconds**. Amar swims the back stroke in 1:07, the breast stroke in 1:09 and has never competed in the free style or butterfly. Akbar is a free style specialist averaging 1:00 for the 100 metres but can also swim the breast stroke in 1:06 and butterfly in 1:12. Antony swims all styles back stroke 1:09, butterfly 1:11, free style 1:05 and breast stroke 1:24. Ashoka swims only the butterfly 1:11. Alex swims the back stroke 1:18, the breast stroke 1:14, free style 1:06 and the butterfly 1:10. Which swimmer should be assigned to which swimming style? Who will not be in the relay?

88. Average time taken by an operator on a specific machine is tabulated below. The management is considering replacing one of the old machines by a new machine and the estimated time for operation by each operator of the new machine is also indicated.

Operator	**Machines**					
	1	**2**	**3**	**4**	**6**	**New**
A	10	12	8	10	10	11
B	9	10	8	7	9	10
C	12	13	14	14	14	11
D	9	9	9	8	10	9
E	7	8	9	9	8	8

d. Find out an allocation of operators to the old machine to achieve a minimum operation time.
e. Reset the problem with the new machine and find out the allocation of the operators to each machine and comment whether is advantageous to replace an old machine to achieve a deduction in operation time only.
f. However will the operators be re-allotted to the machines after replacement?

89. XYZ Airlines operating 7 days a week has given the following time table. Crews must have a minimum layover of 3 hours between flights. Obtain the pairing of flights so that the layover can be kept at the minimum. Also find the base of each crew.

Chennai – Mumbai				Mumbai - Chennai		
Flight No.	Dep	Arr		Flight No.	Dep	Arr
A1	6.15 am	7.45 am		B1	8 am	9.30 am
A2	8.30 am	10.15 am		B2	9.15 am	11 am
A3	2 pm	4 pm		B3	7 pm	9.15 pm

An organization producing 4 different products viz., A,B,C and D having 4 operators viz., P,Q,R and S who are capable of producing any of the four products, works effectively 7 hours a day. The times in minutes required for each operator for producing each of the products are given below in the nine cells of the following matrix with profits in Rs per unit. Find out the assignment of operators which will maximize the profit.

Operator	**Product**			
	A	**B**	**C**	**D**
P	7	8	10	12
Q	8	6	5	6
R	8	8	12	9
S	20	12	15	15
Profit (Rs)	3	4	2	1

90. Five swimmers are eligible to compete in a relay team which is to consist of four swimmers swimming four different swimming styles: Back stroke, Breast stroke, free style and butterfly. The time taken for the five swimmers – Anand, Baskar, Chandra, Darwin and Evan to cover a distance of 100 metres in various swimming styles are given below in **minutes:seconds**. Anand swims the back stroke in 1:08, the breast stroke in 1:05 and has never competed in the free style or butterfly. Baskar is a free style specialist averaging 1:01 for the 100 metres but can also swim the breast stroke in 1:14 and butterfly in 1:20. Chandra swims all styles back stroke 1:10, butterfly 1:12, free style 1:05 and breast stroke 1:22. Darwin swims only the butterfly 1:11. Evan swims the back stroke 1:21, the breast stroke 1:16, free style 1:06 and the butterfly 1:10. Which swimmer should be assigned to which swimming style? Who will not be in the relay?

91. To stimulate interest and provide an atmosphere for intellectual discussion a finance faculty in a management school, decides to hold special seminars on four contemporary topics-leasing, portfolio management, private mutual funds, swaps and options. Such seminars should be held once per week in the afternoons. However, scheduling these seminars (one for each topic and not more than one seminar per afternoon) has to be done carefully so that the number of students unable to attend is kept to a minimum. A careful study indicates that the number of students who cannot attend a

particular seminar on a specific day is as follows:

Days	Leasing	Portfolio Management	Private Mutual Fund	Swaps and Options
Mon	45	45	55	50
Tue	25	40	40	30
Wed	55	58	29	21
Thur	31	32	24	33
Fri	13	19	10	29

Find out the optimum schedule of the seminars. Find out the total number of students who will be missing at least one seminar.

92. The secretary of a school is taking bids on the city's four school bus routes. Four companies have made the bids as detailed in the following table:

Company	Bids in Rupees			
	Route 1	Route 2	Route 3	Route 4
C1	3000	6000	-	-
C2	-	4500	-	6000
C3	2800	-	1950	-
C4	-	-	4500	5000

Suppose each bidder can be assigned only one route. Use the assignment model to minimize the school's cost of running the four bus routes.

93. Find the optimal solution for the following traveling salesman's problem

From	To				
	A	B	C	D	E
A	-	3	4	5	4
B	5	-	9	3	4
C	6	5	-	6	3
D	2	4	3	-	7
E	4	6	5	8	-

94. Find the optimal solution for the following traveling salesman's problem

From	**To**			
	Delhi	**Chandigarh**	**Shimla**	**Jaipur**
Delhi	-	3	5	6
Chandigarh	6	-	4	7
Shimla	8	7	-	3
Jaipur	6	3	7	-

95. Find the optimal solution for the following traveling salesman's problem

From	To				
	A	B	C	D	E
A	-	3	7	5	4
B	5	-	8	3	4
C	6	5	-	7	3
D	2	4	3	-	8
E	4	6	5	7	-

TRANSPORTATION

DEFINITION;
Transportation models deals with problems concerning as to what happens to the effectiveness function when we associate each of a number of origins (sources) with each of a possibility different number of destination (JOBS). The total movement from each origin and the total movement of each destination is given and it is desired to fine how the associations be made subject to the limitations on totals.

ASSUMPTIONS

- Total quantity of item available at different sources is equal to the total requirement at different destination.
- Item can transported conveniently from all the sources to destinations.
- The unit transportation cost of the item from all the sources sto destinations in certainly and precisely known.
- The transportation cost on a given route is directly proportional to the number of units shipper on that route.
- The objective is to minimize the total transportation cost for the organization as a whole and not for individual supply and distribution centres.

DEFINITION
Feasible Solution:
A feasible solution to a transportation problem is a set of non-negative allocations (xii) that satisfies the rim (row and column) restrictions.

Basic feasible solution:
A feasible solution to a transportation problem is said to be a basic feasible solution if it contains no more than m+n-1 on negative allocation, where, m is the number of rows and n is the number of columns of the transportation problem.

Optimal solutions:
A feasible solution (not necessarily basic) that minimizes (maximizes) the transportation cost (profit) is called an optimal solution.

Non-degenerate basic feasible solutions:
A basic feasible solution to a (mxn) transportation problem is said to be non-degenerate if
* The total number of non-negative allocations is exactly m+n-1
* These m+n-1 allocations are independent positions.
Degenerate basics feasible solutions:
A basics feasible solution in which the total number of non-negative allocation is less than m+n-1 is called degenerate basic feasible solutions.

Matrix Technology:
The matrix used in the transportation models consist of squares called cells; which when stacked form "columns" vertically and rows horizontally.

The cells located at the intersection of a row and a column is designated by its row and column headings. Thus the cell located at the intersection of row A and column 3 is called cell (A,3) unit cost are placed in each cell.

North-west corner rule or north west corner method (NCWM);
This rules may be stated as follows;

- Start in the north-west (upper left) corner of the requirements table (ic) the transportation matrix framed in step 1 and compare and supply of plant 1 (call it S1) with requirement of distribution centre 1 (call it D1)
- If D1 cs1 i.e; if the amount required at D1 is less than the number of units available at S1, set X11 equal to D1, fine the balance supply and demand and proceed to cell (1,2) (i.e; proceed horizontally)

Row minima method;
This method consists in allocating as much as possible in the lowest cost cell and then further allocation is done in the cells with second lowest cost and so on. In case of the tie among the costs, select the cell where allocation of more number of units scan be made.

Vogel's approximation method (VAM)
In the transportation matrix, if an allocation is made in the second lowest cost cell insteas of the lowest cell, this allocation will have associated with it has penalty corresponding to the difference of these two costs due to loss of advantage that is to say\ if we compute the difference between the two lowest costs for each row or column. We find the opportunity cost relevant to each row and column.

The stepping stone method;
Consider the matrix giving initial feasible solution for the problems under consideration. Let us start with any arbitrary empty cells (a cell without allocation) say (3,2) and allocate +1 units to this cell. As already discussed, in order to keep up the column 2 restriction -1 must be

Allocated to cells(1,2) and keep up the row 1 restriction, +1 must be allocated to cells(1,1)and consequently -1 must be allocated to cells(3,1).

The Modified Distribution method (MODI) or U-V Method:

In the steeping stone method or closed path in traced for each unoccupied cell. Cell evaluations are found the basic cell. In the method distribution method cell evaluation of all the unoccupied cells are calculated simultaneously and only one closed path for the most negative cell to traced. Thus it provides considerable time saving over the steeping stone method.

TRANSPORTATION ALGORITHM:
Transportation algorithm for a minimization problem as discussed earlier can be summarized in the following steps.

1. Construct the transportation matrix for this enter the supply a; from the orgins, demand bj at the destinations and the unit costs cij in the various cells.
2. Find initial basic feasible solution by Vogel's approximation method or any of the other given methods.
3. Perform optimally test using modified distribution method, for this find dual variables ui and vj. Such that ui+vj=cij for occupied cells starting with, say, v1=D all other variables can be evaluated.
4. Compute the cells evaluated = cij-(ui+vj) for vacant cells, if all cells evaluations are positive or zero, the current basis of feasible solution is optimal. In case any evaluation is negative the current solution is not optimal.
5. Select the vacant cell with the most negative evaluation, this is called identified cell
6. Make as much allocation in the identified cell and possible so that it become basis ie; reallocate the maximum possible number of unit to this cells keeping in mind of conditions. This will make allocation in one basic cell zero and in other basic cells; the location will remain non-negative greater then or equal to zero. The basis cell whose allocation becomes zero will leave the basis.
7. Return to step 3 repeat the process till optimum solution is optained.

The Unbalance Transportation Problem:

In the problem discussed so far in total availability from all the origins was equal to the total demand at all the destination i.e; ∑at t=1 = ∑bj j=1 . Such problems are called balanced transportation problem. In many real life situation; however, the total availability may not be equal to the total demand i.e; ∑at t=1 = ∑bj j=1. Such problem are called unbalanced transportation problem.

PROBLEMS

1. Find the basic solution using the Least Cost Method for the following transportation problem.

Factory	Stores				Demand
	S1	S2	S3	S4	
F1	1	3	4	5	28
F2	2	7	8	3	43
F3	1	4	5	7	37
Supply	27	32	24	25	108

2. Find the basic solution using the Vogel's Approximation Method for the following transportation problem.

Factory	Stores				Demand
	S1	S2	S3	S4	
F1	12	18	24	14	28
F2	19	25	13	6	43
F3	-3	0	21	15	37
Supply	27	32	24	25	108

3. Solve the following transportation problem to find the optimal solution. The values given are profit in Rs.

Location	Warehouse				Demand
	W1	W2	W3	W4	
L1	2	3	1	2	15
L2	1	-2	3	0	35
L3	2	5	18	3	50
Supply	10	20	55	15	100

4. Find the basic solution using the NWC Method for the following transportation problem. The values are profit in Rupees

Factory	Stores				Demand
	S1	S2	S3	S4	
F1	12	18	24	14	28
F2	19	25	13	6	43
F3	-3	0	21	15	37
Supply	27	32	24	25	108

5. Solve the following transportation problem to find the optimal solution. The values given are profit in Rs.

Location	Warehouse				Demand
	W1	W2	W3	W4	
L1	2	3	1	-2	25
L2	1	2	3	0	35
L3	2	5	8	3	40
Supply	20	10	55	15	100

6. Find the basic solution using the Least Cost Method for the following transportation problem.

Factory	Stores				Demand
	S1	S2	S3	S4	
F1	12	18	24	14	28
F2	19	25	13	6	43
F3	-3	0	21	15	37
Supply	27	32	24	25	108

7. Find the basic solution using the Least Cost Method for the following transportation problem.

Factory	Stores				Demand
	S1	S2	S3	S4	
F1	12	18	24	14	28
F2	19	25	13	6	43
F3	-3	0	21	15	37
Supply	27	32	24	25	108

8. Siraj enterprises has three factories at locations, A, B,C which supplies three warehouses located at D, E, F. Monthly factory capacities are 10, 80 and 15 units respectively. Monthly warehouse requirements are 75, 20 and 50 units respectively. Unit shipping cost in Rs. are given below:

Factory	Warehouse		
	D	E	F
A	5	1	7
B	6	4	6
C	3	2	5

The penalty costs for not satisfying demand at the warehouses D, E, and F are Rs.5, Rs.3 and Rs.2 per unit respectively. Determine the optimal distribution for the company.

9. Jennath enterprises has three factories A, B and C which supplies 4 warehouses located at D, E, F and G. Monthly factory capacities are 160, 150 and 190 units respectively. Monthly warehouse requirements are 80, 90,110 and 160 units respectively. Unit shipping cost in Rs. are given below:

Factories	Warehouse				Demand
	D	E	F	G	
A	42	48	38	37	160
B	40	49	52	51	150
C	39	38	40	43	190
Supply	80	90	110	160	

Determine the optimal distribution for the company to minimize the shipping cost.

10. Solve the following transportation problem to find the optimal solution. The values given are profit in Rs.

Location	Warehouse				Demand
	W1	W2	W3	W4	

L1	2	-2	1	1	30
L2	1	0	3	4	30
L3	2	3	8	6	40
Supply	20	20	45	15	100

11. Solve the following assignment problem to find the optimal solution. The values given are profit in Rupees.

Jobs	Workers			
	W1	W2	W3	W4
Job 1	2	-2	1	1
Job 2	1	0	3	4
Job 3	2	3	8	6
Job 4	20	20	45	15

12. Find the optimal solution from the following transportation problem.

Factory	**Stores**				**Demand**
	S1	**S2**	**S3**	**S4**	
F1	12	18	24	14	28
F2	19	25	13	6	43
F3	-3	0	21	15	37
Supply	27	32	24	25	108

13. Find the optimal solution from the following transportation problem.

Location	**Warehouse**				**Demand**
	W1	**W2**	**W3**	**W4**	
L1	10	2	8	5	32
L2	13	15	4	13	48
L3	4	10	5	15	100
Supply	45	55	25	35	

14. Solve the following assignment problem to find the optimal solution.

Jobs	**Workers**				
	W1	**W2**	**W3**	**W4**	**W5**
J1	12	10	15	19	24
J2	12	24	15	32	10
J3	12	15	16	14	13
J4	12	14	18	24	29

15. Solve the following transportation problem to find the optimal solution. The values given are profit in Rs.

Location	**Warehouse**				**Demand**
	W1	**W2**	**W3**	**W4**	
L1	2	3	1	2	15
L2	1	-2	3	0	35
L3	2	5	18	3	50
Supply	10	20	55	15	100

16. Solve the following assignment problem. The values are profit in Rupees

Factory	**Stores**			
	S1	**S2**	**S3**	**S4**
F1	12	18	24	14
F2	19	25	13	6
F3	-3	0	21	15
F4	27	32	24	25

17. Solve the following transportation problem to find the optimal solution.

Location	**Warehouse**				**Demand**
	W1	**W2**	**W3**	**W4**	
L1	2	3	1	-2	25
L2	1	2	3	0	35
L3	2	5	8	3	40
Supply	20	10	55	15	100

18. Suraj enterprises has three factories at locations, A, B,C which supplies three warehouses located at D, E, F. Monthly factory capacities are 10, 80 and 15 units respectively. Monthly warehouse requirements are 75, 20 and 50 units respectively. Unit shipping cost in Rs. are given below:

Factory	**Warehouse**		
	D	**E**	**F**
A	4	1	7
B	7	4	6
C	3	2	5

The penalty costs for not satisfying demand at the warehouses D, E, and F are Rs.5,

Rs.3 and Rs.2 per unit respectively. Determine the optimal distribution for the company.

19. Siraj enterprises has three factories at locations, A, B,C which supplies three warehouses located at D, E, F. Monthly factory capacities are 10, 80 and 15 units respectively. Monthly warehouse requirements are 75, 20 and 50 units respectively. Unit shipping cost in Rs. are given below:

Factory	Warehouse		
	D	E	F
A	5	1	3
B	6	4	2
C	3	2	5

The penalty costs for not satisfying demand at the warehouses D, E, and F are Rs.5, Rs.3 and Rs.2 per unit respectively. Determine the optimal distribution for the company.
Note: Blank space means the manager does not prefer that particular room.

20. Unit shipping cost in rupees are given below, determine the optimal distribution for the company to minimize the shipping cost.

Factories	Warehouse				Demand
	D	E	F	G	
A	2	4	1	7	120
B	4	9	2	5	150
C	3	3	0	3	190
Supply	120	100	120	160	

21. Solve the following transportation problem to find the optimal solution.

Location	Warehouse				Demand
	W1	W2	W3	W4	
L1	2	3	1	-2	25
L2	1	2	3	0	35
L3	2	5	8	3	40
Supply	20	10	55	15	100

22. Max enterprises has three factories A, B and C which supplies 4 warehouses located at D, E, F and G. Monthly factory capacities are 160, 150 and 190 units respectively. Monthly warehouse requirements are 80, 90,110 and 160 units respectively. Unit shipping cost in Rs. are given below:

Factories	Warehouse				Demand
	D	E	F	G	
A	23	24	54	21	160
B	12	13	15	18	150
C	3	9	8	4	190
Supply	80	90	110	160	

Determine the optimal distribution for the company to minimize the shipping cost.

23. Unit shipping cost in rupees are given below, determine the optimal distribution for the company to minimize the shipping cost.

Factories	Warehouse				Demand
	D	E	F	G	
A	2	4	3	7	160
B	4	9	2	5	250
C	3	3	0	3	190
Supply	170	150	120	160	

24. Solve the following transportation problem to find the optimal solution.

Location	Warehouse				Demand
	W1	W2	W3	W4	
L1	2	3	1	-2	25
L2	1	-2	3	0	35
L3	10	8	5	3	40
Supply	20	10	55	15	100

25. Solve the following transportation problem.

Locati	Warehouse	Dema

on	W1	W2	W3	W4	nd
L1	5	8	2	10	32
L2	13	12	4	13	48
L3	4	10	5	15	50
Supply	45	55	25	35	

26. Solve the following transportation problem.

Location	Warehouse				Demand
	W1	W2	W3	W4	
L1	5	8	2	10	32
L2	10	12	4	13	48
L3	4	10	5	11	50
Supply	45	55	25	35	

27. Find the basic solution using the Least Cost Method for the following transportation problem.

Factory	Stores				Demand
	S1	S2	S3	S4	
F1	1	3	4	5	28
F2	2	7	8	3	43
F3	1	4	5	7	37
Supply	27	32	24	25	108

28. Solve the following traveling salesman problem to find the optimal solution.

From	To			
	W1	W2	W3	W4
W1	-	-2	1	1
W2	1	-	3	4
W3	2	3	-	6
W4	6	2	5	-

29. Nisha enterprises has three factories A, B and C which supplies 4 warehouses located at D, E, F and G. Monthly factory capacities are 160, 150 and 190 units respectively. Monthly warehouse requirements are 80, 90,110 and 160 units respectively. Unit shipping cost in Rs. are given below:

Factories	Warehouse				Demand
	D	E	F	G	
A	20	10	15	25	160
B	42	48	38	37	150
C	40	49	52	51	190
Supply	80	90	110	160	

Determine the optimal distribution for the company to minimize the shipping cost.

30. Find the optimal solution from the following transportation problem.

Location	Warehouse				Demand
	W1	W2	W3	W4	
L1	10	2	8	5	32
L2	13	15	4	13	48
L3	4	10	5	15	100
Supply	45	55	25	35	

31. Find the optimal solution from the following transportation problem.

Factory	Stores				Demand
	S1	S2	S3	S4	
F1	-3	18	24	14	28
F2	12	25	15	6	43
F3	19	0	21	15	37
Supply	27	32	24	25	108

32. Find the optimal solution from the following transportation problem. The values given are profit in Rs.

Location	Warehouse				Demand
	W1	W2	W3	W4	
L1	5	8	2	10	32
L2	6	-	4	13	48
L3	4	10	-5	15	50
Supply	45	55	25	35	

33. Find the optimal solution from the following transportation problem.

Location	Warehouse				Demand
	W1	W2	W3	W4	
L1	21	12	13	31	15
L2	11	22	21	10	35
L3	18	15	5	3	30
Supply	20	20	20	20	

34. Find the optimal solution from the following transportation problem.

Location	Warehouse				Demand
	W1	W2	W3	W4	
L1	10	2	8	5	62
L2	13	15	4	13	48
L3	4	10	5	15	100
Supply	45	85	25	35	

35. Find the basic solution from the following transportation problem.

Location	Warehouse				Demand
	W1	W2	W3	W4	
L1	6	18	0	13	15
L2	12	9	11	17	35
L3	10	12	14	16	30
L4	10	2	8	12	10
Supply	20	20	20	30	

36. Find the basic solution using the Least Cost Method for the following transportation problem.

Factory	Stores				Demand
	S1	S2	S3	S4	
F1	4	-	5	8	28
F2	-3	0	21	15	43
F3	12	18	24	14	37
Supply	27	32	24	25	108

37. Find the basic solution using the Least Cost Method for the following transportation problem.

Factory	Stores				Demand
	S1	S2	S3	S4	
F1	1	2	5	3	28
F2	-	0	-4	6	33
F3	2	5	3	2	47
Supply	27	32	24	25	108

38. Solve the following transportation problem.

Location	Warehouse				Demand
	W1	W2	W3	W4	
L1	5	8	2	10	12
L2	13	12	4	13	23
L3	4	10	5	15	25
Supply	10	18	14	18	

39. Find the optimal solution from the following transportation problem.

Location	Warehouse				Demand
	W1	W2	W3	W4	
L1	5	8	2	10	32
L2	13	12	4	13	48
L3	4	10	5	15	50
Supply	45	55	25	35	

40. Solve the following transportation problem.

Location	Warehouse				Demand
	W1	W2	W3	W4	
L1	1	3	4	8	32
L2	9	8	7	1	48
L3	-1	5	6	2	50
Supply	45	55	25	35	

41. Find the basic solution using the Vogel's Approximation Method for the following transportation problem.

Factory	Stores				Demand
	S1	S2	S3	S4	
F1	12	18	24	14	28
F2	19	25	13	6	43
F3	-3	0	21	15	37
Supply	27	32	24	25	108

42. Find the optimal solution from the following transportation problem.

Location	Warehouse				Demand
	W1	W2	W3	W4	
L1	21	12	13	31	15
L2	11	22	21	10	35
L3	18	15	5	3	30
Supply	20	20	20	20	

43. Solve the following transportation problem to find the optimal solution. The values given are profit in Rupees.

Location	Warehouse				Demand
	W1	W2	W3	W4	
L1	12	3	1	-2	25
L2	10	2	3	0	35
L3	2	5	8	3	40
Supply	20	10	55	15	100

44. Unit shipping cost in rupees are given below, determine the basic solution using VAM.

Factories	Warehouse				Demand
	D	E	F	G	
A	2	4	3	7	160
B	4	9	2	5	150
C	3	3	0	3	190
Supply	120	100	120	160	

45. Solve the following transportation problem.

Location	Warehouse				Demand
	W1	W2	W3	W4	
L1	5	8	2	10	32
L2	13	12	4	13	48
L3	4	10	5	15	50
Supply	45	55	25	35	

Unit – IV
NETWORK ANALYSIS

OBJECTIVES OF NETWORK ANALYSIS

- To complete the project within the stipulated period.
- Optimum utilization of the available resources.
- Minimization of cost and time required for completion of the period.
- Minimization of idle resources and investment in inventory.
- To identify the bottlenecks, if any and to focus attention on critical activities.
- To reduce the setup and change over costs.

ADVANTAGES OF NETWORK ANALYSIS:

- They are most valuable and powerful for planning, scheduling and control of operation in large and complex projects.
- They are useful tools to evaluate the level of performance by comparing actual performance against the planned targets.
- They help to determine for proper integration and co-ordination of various operations.
- These techniques help the management in achieving the objectives with minimum of time and least cost and also in predicting the probable project duration and the associated cost.
- Application of PERT and CPM have resulted in saving of time with directly results in saving of cost saving in time or easy completion of the project results in earlier return of revenues and introduction of the competition resulting in increased profits.
- Application of network techniques has resulted in better managerial control, improved utilization of resources, improved communication and progress reporting and better decision making.

LIMITATIONS:

- Constructions of networks for complex projects are complicated and time consuming due to trial and error approach.
- Estimation of reliable and accurate duration of activities is a difficult exercise.
- With too many resources constraints the analysis becomes very difficult.
- Time cost tradeoff procedure in many situations is complicated.

DIFFICULTIES USING IN NETWORK METHODS:

- Difficulty in securing the realistic time estimates. In the case of new and non-repetitive type projects, the time estimates are often more guesses.
- The tendency to oppose changes results in the difficulty of persuading the management to accept these techniques.
- The planning and implementation of the network require personnel trained in the network methodology, managements are reluctant to spare the existing staff to learn these techniques or a recruit trained person.

➢ Developing a clear logical network is also troublesome. It depends upon the data input and thus the personnel who provide the data.

APPLICATIONS OF NETWORK TECHNIQUES:

- ✓ Construction industry
- ✓ Manufacturing
- ✓ Maintenance planning
- ✓ Administration
- ✓ Marketing
- ✓ Inventory planning
- ✓ Other areas of applications.

DISTINCTION BETWEEN PERT AND CPM:

❖ CPM is oriented activity, a cpm network is built on the basis of activities also results of various calculations are considered in terms of activities of the projects, and on the other hand PERT is event oriented.

❖ CPM is a deterministic model, it does not take into account the uncertainties involves in the estimation of time for the execution of an activity, each activity is assigned a single time based on past experience.

❖ CPM places dual emphasis on project time as well as cost and finds the trade off between project time and project cost. By our playing additional resources, it helps to manipulate project duration within the certain limits so that project duration can be reduced at optimum cost. On the other hand PERT is primarily concerned with time only. It helps to schedule and co-ordinate various activities so that project can be completed in schedule time.

❖ CPM is primarily user for projects which are repetitive in nature and comparatively small in size. PERT is generally used for projects where time refused to complete the activities is not known a priority. Thus PERT is uses for large one time research and development type of project.

Programme Evaluation and Review Technique:

The PERT system is based on three time estimates of the performance time of an activity they are:

1. The optimistic time estimate:

the shortest possible required for the completion of an activity.

2. The pessimistic time estimation:

the maximum possible time the activity will take if every thing goes bad.

3. The most likely time estimated:

the time an activity will take if executed under normal conduction. It is the model value.

Cost Analysis and Crashing the Network:

1. Project cost: Cost which is involved in project, while consuming in resources.

2. Direct cost: Direct amount spend is execution of the individual activity.

Crash time:

It is thus the minimum activity duration to which an activity can be compressed by increasing the resources and hence by increasing the direct cost.

Indirect cost:

- Fixed cost
- Variable cost

Fixed cost:

Fixed cost is the cost does not change with duration of time. For example Administration expenses, Rent, Taxes etc.

Variable cost:

Variable cost is the cost affected by the duration of the project. This consists of Overhead expenses like Supervision charges, interest, Depreciation etc.

Total cost:

Direct cost + Indirect cost

Crashing (Contracting and Compressing) the Network:

The normal time for the completing of the project will be adding the direct and indirect costs duration of the critical activities and normal direct cost of the project will be the sum of the normal cost of the activity since each and every activities has to be executed to complete the project.
The minimum time that the project will take for its completion will be sum of the crashed time durations of the activities along the critical path if all the activities are crashed the cost will be very high without any additional advantage over and above the one obtained by crashing only the critical activities therefore the non-critical activities need not be expedited since their crashing is not going to decrease the project duration.

CRITICAL PATH ANALYSIS:

The critical path of a network gives the shortest time in which the whole project can be completed. It is chain of activities with the longest time duration. They are critical in the sense that delays in any of them results in the delay of completion of the project. There may be more than one critical path in the network and it is possible for the critical path to run through a dummy. It consists of the following steps:

1.Calculate the time schedule for the activity:

It involves the determination of the time by which an activity must begin and time it should be completed. Time schedule data for each activity includes the calculation at the earliest start , earliest finish , latest start and finish and the float.

2.Calculate the time schedule for the completion of the entire project:

Critical activities are the ones which must be started and completed on time. The path containing these activities is the critical path and is the longest path in terms of duration.

TYPES OF FLOAT:

1.Total Float:

Difference between the max. time available to perform the activity and the activity duration. The maximum time available in the from the earliest start time to the latest completion time. Thus for an activity i-j having duration tij

Max. Time available = L – E
Total Float = L- E – tij

2.Free Float :

It is that portion of the total float in which an activity can be manipulated without affecting the floats of the subsequent activities.

Free Float = i-j = T.F.-(L- E) of the event j.

3. Independent Float:

It is that portion of the total float within which an activity can be delayed for start without affecting the floats of proceeding activities. It is computed by subtracting the tail event stack from free float. If the result is negative, it is taken as zero.

i-j= F.F – (L-E) of tail event j.

OTHER FLOATS:

- Interfering Float:

It is difference between latest finish time and the earliest start time of the activity under consideration and the earliest start time of the following activity, or zero, whichever is larger.
Interfering Activity = T.F. – F.F.

- Sub – critical Activity:

An activity having higher float than the critical activity is called sub- critical activity and demands normal attention but allows some freedom of action. A network can may have more than one sub – critical path.

- Super critical Activity:

An activity having negative float is called a super critical activity. Such an activity demands very special attention and action.

- Slack:

It is the time by which occurrence of an event can be delayed. It is the difference between latest and the earliest occurrence of time.
S = L –E of the event

DEFINE PROJECT? WHAT ARE ITS CHARCTERISITICS?

A Project is a combination of inter related activities which must be executed in a certain order before the entire task is completed.

Characteristics of a Project:

1. A project has identifiable a beginning and end points. It is entity by itself.
2. It is not a permanent entity.
3. It is to be completed at a target date.
4. The objectives are clear and output is product definite
5. It is usually large and complex with time horizon of 2 to 3 years. However , some projects have taken more than 10 yrs , while others have lasted for less than 6 months.
6. It involves huge investment.

WHAT IS PROJECT PLANNING? WHAT ARE STEPS INVOLVED IN PROJECT PLANNING.

Project planning is an important phase during which are the set plans and strategies of project execution , keeping in mind the policies, procedures and rules of the organization. It has two important aspects – identification of the activities to be performed and the estimation of the required resources

STEPS INVOLVED IN PROJECT PLANNING.

1. Setting the objectives of the project and the assumptions to be made.

2.Determination of tome estimates of these activities.
3.Estimation of the resources.
4.Study of alternative ways of attaining the goals.
5.Establishment of inter – dependence relationship between the activities i.e the sequence of performing the activities.
6.Developing the breakdown structure. Depending upon the objectives of the management, the availability of computational aids, the project is broken down into clearly definable activities.

WHAT IS PROJECT SCHEDULING

Scheduling is the laying out of the project activities along a time sequence in which they are to be performed so as to assign the starting and finishing dates to various activities and to allocate resource to them. In other words, scheduling is the preparation of the time table of the implementation of the activities and computation of resources required at different stages of time. The corrective measures to rectify the deviations from the plan should be suggested. This requires decision – making with regard to scheduling of resources, scheduling of jobs, crashing of the projects.

WHAT IS WORK BREAKDOWN STRUCTURE

A Project is a combination of inter related activities which must be executed in a certain order for its completion. The process of dividing the project into these activities is called WORK BREAKDOWN STRCTURE

WHAT ARE BASIC TOOLS AND TECHINQUES OF PROJECT MANAGEMENT?

1.Bar charts, milestone charts and velocity diagram.
2.Network techniques.

<u>WHAT IS PROJECT CONTROLLING? WHAT ARE STEPS INVVOLVED</u>

Controlling phase is the follow up of the planning and scheduling phase. While planning and scheduling are undertaken before the actual execution of the project.

The project is monitored to find deviations in actual progress from scheduled plan and to apply the corrective measures, so as to achieve the targets.

<u>The involves the following steps:</u>

1.Setting standards and targets with regard to time and cost of the project.
2.Reviewing the progress by the work accomplished to work, scheduled at different stages of time and finding the deviations.
3.Evaluating the efforts of deviations on the project plan.
4.Updating the project schedule.

<u>BAR CHARTS, MILESTONE CHARTS & VELOCITY DIGRAM:</u>

Bar charts are two dimensional pictorial representation of the project. In a bar chart, the activities of the project are shown on the axis and their durations are represented on the other axis. A bar chart helps to

review the project progress, allows for scheduling review the project and highlights the critical activities and other bottlenecks in the completion of the project.

Milestone charts are improved versions of Bar charts. However , milestone charts represents the events which mark either mark beginning or end of the activity.

Velocity diagrams are useful for representing the activities which require a series of crew working in a given sequence.

NETWORK TECHNIQUES;

A Network is a symbolic representation of the essential characteristics of a project. PERT AND CPM is two mostly widely applied techniques.

(a) **PROGRAMME EVALUTION AND REVIEW TECHNIQUE (PERT):**

It uses event oriented network in which successive events are joined by arrows. It is preferred for projects that are non – repetitive and which time for various activities cannot be precisely pre – determined.

Launching a new product in the market by a company, research and development of a new war weapon. Launching of satellite, sending space craft are PERT projects.

(b) **CRITICAL PATH METHOD:**

It uses activity oriented network which consists of a number of well recognized jobs, tasks or activities. Each activity is represented by arrow and the activities are joined together by events.

CPM is generally used for simple, repetitive types of projects for which the activities times and costs are certainly and precisely known.

Projects like construction of a building road. Bridges, yearly closing of accounts are handled by CPM.

RESOURCE SCHEDULING

During the development of PERT AND CPM we have generally assumed that sufficient resources are available to perform the various activities. At a certain time the demand on the particular resources is the cumulative demand of that resource on all activities being performed at that time. Going according to developed plan, the demand of a particular resource may fluctuate from very high at one time to very low at another time. If it is a material or unskilled labour which has to be procured from time to time, the fluctuation in the demand will not affect the cost of the project. Depending upon the type of constrains the resource scheduling situation may be of 2 types:

1. The constraint may be the total project duration:

In this case the resource scheduling situation only smoothens the demand on resources in order that the demand of the resource is uniform as possible.

2. The second type of constraint may be on availability of certain resources:

Here, the project duration is not treated as an invariant, but the demand on certain specified resources should not go beyond the specified level. The operation of resource scheduling is called "Resource leveling".

Resource Smoothing:

The first step in resource smoothing is to determine the maximum requirement. One way is to draw the time scaled version of the network and assign resource requirement to activities. Then, below the time scaled network, the cumulative requirements for each time are plotted. The result is a "Load Histogram". The Load histogram which is also known as force curve may be plotted on the basis of early times or the late start times of the activities. This load histogram establishes the frame work within which smoothing must occur.

Resource Levelling:

In the load smoothing operation, the duration of the project was kept constant and the non critical activities were shifted within the available floats to smoothen the resource requirements. In resource leveling, the constraint is on the availability of the resources while the project duration can be extended. In this method, the activities are so rescheduled that the maximum requirement of resources does not cross their availability. However the available resources should not be less that the maximum quantity required by any activity of the project. These types of resource leveling procedures are usually followed.

1. Variable resource leveling:

As already discussed, the requirement of resources over the project duration is usually non – uniform. [fig.a] . To make it uniform , the usual procedure is to engage a reasonable number of workers at the beginning and then to increase it gradually, reaching a peak near the end of the project. Once the maximum is reached, there is gradual reduction of the work force.

2. Fixed Resource Levelling:

Here, the number of worker remaining constant throughout the project duration. As the requirement is highly fluctuating,there can be both excessive overtime as well as idle work force at different times. The aim,here,is to select the optimum number of workmen so that both overtime as well as unproductive standby persons are minimized.

3. Combined Resource Levelling:

Here, fixed no of person are initially engaged and to satisfy the increasing needs, the number is increased in accordance with the pattern of variable resource leveling.

PROBLEMS

1. From the following
   a. Draw the network and identify all the paths through it.
   b. Find the expected duration and variance for each activity.
   c. Calculate the variance and Standard deviation of the project
   d. What is the probability that the project will be completed within (i.) 25 weeks (ii.) 19 weeks (iii.) The number of days for a confidence level of 91%.

Activity	1-2	2-3	1-3	2-4	3-5	4-5	5-6
a	2	3	2	1	4	2	3
m	1	4	3	1	6	6	6
b	6	11	10	7	14	10	15

2. The following table gives data on normal time and cost and crash time and cost for a project.

Activity	Normal Time (weeks)	Normal Cost (Rs.)	Crash Time (weeks)	Crash Cost (Rs.)
1-2	3	300	2	400
2-3	3	30	3	30
2-4	7	420	5	580
2-5	9	720	7	810
3-5	5	250	4	300
4-5	0	0	0	0
5-6	6	320	4	410
6-7	4	400	3	470
6-8	13	780	10	900
7-8	10	1000	9	1200

Indirect cost Rs. 50 per week.

a. Draw the network and identify the critical path with a double line.
b. What are the normal project duration and the associated cost?
c. Crash the relevant activities systematically and determine the optimal project completion time and cost.

3. From the following
   - Draw the network and identify all the paths through it.
   - Find the expected duration and variance for each activity.
   - Calculate the variance and Standard deviation of the project
   - The number of days for a confidence level of 95%.

Activity	A	B	C	D	E	F	G
Predecessor	-	-	A,B	B	C,D	B	E,F
a	1	3	5	5	6	6	7
m	3	4	5	6	7	8	9
b	5	5	6	7	7	9	10

4. From the following
   - Draw the network and identify all the paths through it. Find the EST, EFT, LST and LFT.
   - Find out the Total Float, Free Float and Independent Float.

Activity	A	B	C	D	E	F	G	H	I	J
Predecessor	-	A	A	B	B	C	C	E,F	D	G
Duration	2	8	10	6	3	3	7	5	2	8

5. A project consisting of 12 activities is to be analysed by using PERT. The following information is given (time estimates are in days)

Activity	A	B	C	D	E	F	G	H	I	J	K	L
Proceeding Activity	-	-	A	A	B	B	C,D	E	C,D	G,H	F	J,K
Optimistic Time	2	1	4	2	2	5	3	2	3	1	4	2
Normal Time	2	3	7	5	6	9	6	6	5	3	8	5
Pessimistic Time	2	7	8	7	9	11	8	9	8	4	11	7

Construct the PERT network. Indicate the expected total slack for each activity and hence indicate the average critical path. Within how many days would you expect the project to be completed with 99% chance?

6. A project consisting of 12 activities is to be analyzed by using PERT. The following information is given (time estimates are in days)

Activity	A	B	C	D	E	F
Proceeding Activity	-	-	-	C	B	D
Duration	6	3	2	2	1	1

   a. Find the critical path and the project duration.
   b. Find the maximum number of persons required for the project.
   c. If only 6 persons are available find the project duration.
   d. If only 5 persons are available find the project duration.

7. From the following. Draw the network and identify all the paths through it. Find the expected duration and variance for each activity. Calculate the variance and Standard deviation of the project. What is the probability that the project will be completed within (i.) 25 weeks (ii.) 19 weeks (iii.) The number of days for a confidence level of 91%.

Activity	1-2	2-3	1-3	2-4	3-5	4-5	5-6
a	2	3	2	1	4	2	3
m	1	4	3	1	6	6	6
b	6	11	10	7	14	10	15

8. The following table gives data on normal time and cost and crash time and cost for a project.

Activity	Normal Time (weeks)	Normal Cost (Rs.)	Crash Time (weeks)	Crash Cost (Rs.)
1-2	3	300	2	400
2-3	3	30	3	30
2-4	7	420	5	580
2-5	9	720	7	810
3-5	5	250	4	300
4-5	0	0	0	0
5-6	6	320	4	410

6-7	4	400	3	470
6-8	13	780	10	900
7-8	10	1000	9	1200

Indirect cost Rs. 50 per week.

- Draw the network and identify the critical path with a double line.
- What are the normal project duration and the associated cost?
- Crash the relevant activities systematically and determine the optimal project completion time and cost.

9. From the following
   - Draw the network and identify all the paths through it.
   - Find the expected duration and variance for each activity.
   - Calculate the variance and Standard deviation of the project
   - The number of days for a confidence level of 95%.

Activity	A	B	C	D	E	F	G
Predecessor	-	-	A,B	B	C,D	B	E,F
a	1	3	5	5	6	6	7
m	3	4	5	6	7	8	9
b	5	5	6	7	7	9	10

10. From the following
    - Draw the network and identify all the paths through it. Find the EST, EFT, LST and LFT.
    - Find out the Total Float, Free Float and Independent Float.

Activity	A	B	C	D	E	F	G	H	I	J
Predecessor	-	A	A	B	B	C	C	E,F	D	G
Duration	2	8	10	6	3	3	7	5	2	8

11. A project consisting of 12 activities is to be analysed by using PERT. The following information is given (time estimates are in days)

Activity	A	B	C	D	E	F	G	H	I	J	K	L
Proceeding Activity	-	-	A	A	B	B	C,D	E	C,D	G,H	F	J,K
Optimistic Time	2	1	4	2	2	5	3	2	3	1	4	2
Normal Time	2	3	7	5	6	9	6	6	5	3	8	5
Pessimistic Time	2	7	8	7	9	11	8	9	8	4	11	7

Construct the PERT network. Indicate the expected total slack for each activity and hence indicate the average critical path. Within how many days would you expect the project to be completed with 99% chance?

12. A project consisting of 12 activities is to be analyzed by using PERT. The following information is given (time estimates are in days)

Activity	A	B	C	D	E	F
Proceeding Activity	-	-	-	C	B	D
Duration	6	3	2	2	1	1

- Find the critical path and the project duration.
- Find the maximum number of persons required for the project.
- If only 6 persons are available find the project duration.
- If only 5 persons are available find the project duration.

13. From the following
- Draw the network and identify all the paths through it.
- Find the expected duration and variance for each activity.
- Calculate the variance and Standard deviation of the project
- What is the probability that the project will be completed within (i.) 25 weeks (ii.) 19 weeks (iii.) The number of days for a confidence level of 91%.

Activity	1-2	2-3	1-3	2-4	3-5	4-5	5-6
a	2	3	2	1	4	2	3
m	1	4	3	1	6	6	6
b	6	11	10	7	14	10	15

14. The following table gives data on normal time and cost and crash time and cost for a project.

Activity	Normal Time (weeks)	Normal Cost (Rs.)	Crash Time (weeks)	Crash Cost (Rs.)
1-2	3	300	2	400
2-3	3	30	3	30
2-4	7	420	5	580
2-5	9	720	7	810
3-5	5	250	4	300
4-5	0	0	0	0
5-6	6	320	4	410
6-7	4	400	3	470
6-8	13	780	10	900
7-8	10	1000	9	1200

Indirect cost Rs. 50 per week.
- Draw the network and identify the critical path with a double line.
- What are the normal project duration and the associated cost?
- Crash the relevant activities systematically and determine the optimal project completion time and cost.

15. From the following
    - Draw the network and identify all the paths through it.
    - Find the expected duration and variance for each activity.
    - Calculate the variance and Standard deviation of the project
    - The number of days for a confidence level of 95%.

Activity	A	B	C	D	E	F	G
Predecessor	-	-	A,B	B	C,D	B	E,F
a	1	3	5	5	6	6	7
m	3	4	5	6	7	8	9
b	5	5	6	7	7	9	10

16. From the following
    - Draw the network and identify all the paths through it. Find the EST, EFT, LST and LFT.
    - Find out the Total Float, Free Float and Independent Float.

Activity	A	B	C	D	E	F	G	H	I	J
Predecessor	-	A	A	B	B	C	C	E,F	D	G
Duration	2	8	10	6	3	3	7	5	2	8

17. A project consisting of 12 activities is to be analysed by using PERT. The following information is given (time estimates are in days)

Activity	A	B	C	D	E	F	G	H	I	J	K	L
Proceeding Activity	-	-	A	A	B	B	C,D	E	C,D	G,H	F	J,K
Optimistic Time	2	1	4	2	2	5	3	2	3	1	4	2
Normal Time	2	3	7	5	6	9	6	6	5	3	8	5
Pessimistic Time	2	7	8	7	9	11	8	9	8	4	11	7

Construct the PERT network. Indicate the expected total slack for each activity and hence indicate the average critical path. Within how many days would you expect the project to be completed with 99% chance?

18. A project consisting of 12 activities is to be analyzed by using PERT. The following information is given (time estimates are in days)

Activity	A	B	C	D	E	F
Proceeding Activity	-	-	-	C	B	D
Duration	6	3	2	2	1	1

    - Find the critical path and the project duration.
    - Find the maximum number of persons required for the project.
    - If only 6 persons are available find the project duration.
    - If only 5 persons are available find the project duration.

19. From the following

- Draw the network and identify all the paths through it.
- Find the expected duration and variance for each activity.
- Calculate the variance and Standard deviation of the project
- What is the probability that the project will be completed within (i.) 25 weeks (ii.) 19 weeks (iii.) The number of days for a confidence level of 91%.

Activity	1-2	2-3	1-3	2-4	3-5	4-5	5-6
a	2	3	2	1	4	2	3
m	1	4	3	1	6	6	6
b	6	11	10	7	14	10	15

20. The following table gives data on normal time and cost and crash time and cost for a project.

Activity	Normal Time (weeks)	Normal Cost (Rs.)	Crash Time (weeks)	Crash Cost (Rs.)
1-2	3	300	2	400
2-3	3	30	3	30
2-4	7	420	5	580
2-5	9	720	7	810
3-5	5	250	4	300
4-5	0	0	0	0
5-6	6	320	4	410
6-7	4	400	3	470
6-8	13	780	10	900
7-8	10	1000	9	1200

Indirect cost Rs. 50 per week.

- Draw the network and identify the critical path with a double line.
- What are the normal project duration and the associated cost?
- Crash the relevant activities systematically and determine the optimal project completion time and cost.

QUEUING MODELS

INTRODUCTION:

The queuing theory or waiting line theory owes its development to A.K.Erlang. Business of all types like industrial , hospitals, cafeterias, book stores, librarian, banks, post office, petrol pumps, theaters, school all have queuing problem.

Waiting line problems arise either because

1. There is too much demand on the facilities so that we say that there is an excess of waiting time or inadequate number of service facilities.
2. There is too less demand, in which case there is too much idle facility time or too many facilities.

In either cases the problem is to either schedule arrivals or provide proper number of facilities or both so as to obtain the optimum balance between the cost associated with waiting time and idle time.

APPLICATIONS OF QUEUING MODEL:

Waiting line theory is also widely used in manufacturing units. It has been popularly used in the area of tool cribs. There is a general complaint from the foreman that their workmen wait too long in line for tolls and parts. Though the management wants to reduce the overhead charges engaging more attendants can actually reduce overall manufacturing cost, since the workers will be working instead of standing in line.

Queuing theory has also been applied for the solution of problem such as:

1) scheduling a mechanical transport fleet
2) scheduling distribution of scarce war material
3) scheduling of jobs in production control
4) minimization of congestion due to traffic delay at tool booths
5) solution of inventory control problems

ELEMENTS OF A QUEUING SYSTEMS

1) Arrival Distribution

It represents the pattern in which the number of customers arrive at the service facility.
Arrival may be represented by the inter-arrival time, which is period between two successive arrival. Arrivals maybe separated by equal interval of time or by unequal but definitely known intervals of time or unequal intervals of time those probabilities are known these are called random arrivals.

The rate at which customers arrive to be served i.e. number of customers arriving per unit of time is called arrival rate. Mean value of arrival rate is represented by / .

2) Output or departure (service) distribution

It represents the pattern in which the number of customers leave the service facility.
Service time maybe constant or variable but known or random . management scientists have found that they are best described by exponential probability distribution . the rate at which one service channel can perform the service i.e. the number of customer served per unit of time is called service rate. Mean value of service rate is represented by μ.

3) **Service channels**

The queuing system may have a single service channel. The system may have a number of service channel which may be arranged in parallel or in series or in a complex combination of both. A queuing model is called one serve model, when the system has one server only and a multi server model when the system has a number of parallel channels each with one server.

4) Service discipline

Service discipline or order of service is the rule by which customers are selected from the queue for the service. The most common discipline is "first come, first served" according which customers are served in the order of the arrival.

The other discipline is "last come, first served" in the case of big godowns where the items arriving last are taken out first. Still other discipline include random and priority.

Priority is said to occur when an arriving customer is chosen for service ahead of some other customer already in the queue.

5) Maximum number of customer allowed in the system

Maximum number of customers allowed in the system can be either finite or infinite. In some facilities only a limited number of customers are allowed in the system. And new arriving customers are not allowed to join the system unless the number becomes less than the limiting value.

6) Calling source or population

The arrival pattern of the customers depends upon the source which generates them. If there are only a few potential customers the calling source is called finite. If there are large number of potential customers it is usually said to be infinite. There is still another rule for categorizing the source as finite or infinite. The finite source exists when arrival affects the probability of arrival of potential future customers. An infinite source is said to exist when the arrival of a customer does not affect the rate of potential future customer.

7) Customer behaviour

The customer's behaviour is also very important in the study of queue. If a customer decides not to enter the queues since it is too long he is said to have balked. If a customer enters the queue, but after sometime loses patience and leaves it he is said to have reneged the queue. When there are two or more parallel queues and the customers move from one queue to the other they are said to be jockeying.

OPERATING CHARACTERSTICS OF A QUEUING SYSTEM:

Analysis of a queuing system involved a study of its different operating characteristics.

- Queue length (I) = the average number of customer in the queue waiting to get service. This excludes the customers being served.
- System length (L) = the average number of customers in the system including those waiting as well as those being served.
- Waiting time in the queue (W) = the average time for which the customer has to wait in the queue to get the service.
- Total time in the system (W) = the average time spent by the customer in the systems from the moment he arrives till he leaves the systems. It is taken to waiting time plus service time.
- Utilizations factor (P) = It is the proportion of time a server actually spends with the customers. It is also called traffic intensity.

AN EXPLANTORY NOTE ON THE QUEUING FORMULAE:

1) **Traffic intensity:**

The ratio is called the traffic intensity or the utilization factor and it determine the degree to which the capacity of the service station is utilized. For instance, if customer arrives at the rate of 9 per hour and the service rate is 10 per hour, the utilization of service facility is 9/10 = 90%

2) **Average length of the queue:**

$$(\lambda\text{^}2)/\mu(\mu-\lambda)$$

3) **Average length of the system:**

$$\lambda/(\mu-\lambda)$$

4) **Average waiting time in a queue:**

$$\lambda/\mu(\mu-\lambda)$$

5) **Average waiting time in the system:**

$$1/(\mu-\lambda)$$

6) **Capacity Utilisation:**

$$\lambda/\mu$$

7) **Idle time Utilisation:**

$$1 - \frac{\lambda}{\mu}$$

ASSUMPTION OF QUEUING MODELS:

1. The customer arrives for service at a single facility at random according to Poisson distribution with the mean arrival rate.
2. The service time has exponential distribution with the mean service rate
3. The service discipline followed is first come, first served.
4. Customer behaviors is normal i.e., customer desiring service join the queue, wait for their turn and leave only after getting serviced , they do not resort to balking, reneging or jockeying.
5. Service facility behaviour is normal. It serves the customer continuously, without break, as long as there is queue. Also it serves only one customer at a time.
6. The waiting space available for customer in the queue is infinite.
7. The calling sources (population) have infinite size.
8. The elapsed time since the start of the queue is sufficient long so that the system has attained a steady state or stable state.
9. The mean arrival rate is less than the mean service rate.

LIMITATION OF QUEUING MODEL:

1. The waiting space for the customer is usually limited.
2. The arrival time rate may be state dependent. An arriving customer, on seeing a long queue, may not join it and go away without getting service.
3. The arrival process may not be stationery. There may be peak period and slack period during which the arrival rate may be more or less than the average arrival rate.
4. The population of customers may not infinite and the queuing discipline may not be first come, first served.
5. Service may not be rendered continuously. The service facility may breakdown, also the service may be provided in batches rather than individually.
6. The queuing system may not have reached steady state. It may be , instead, in transient state. It is commonly so when the queue just starts and the elapsed time is not sufficient.

CLASSIFICATION OF QUEUING MODEL:

(A) PROBABILISTIC QUEUING MODELS:

1) MODEL I (ERLANG MODEL):

This model is systematically represented by (MM/1: FCFS/∞/∞). This represents poison arrival (exponential interval) poison departure, single server, discipline, infinite population. Since the poison and the exponential distributions are related to each other,

both of them are related to each other, both of them are related by the symbol "m" due to Marko vial property of exponential distributions.

2) MODEL II (GENERAL ERLANG):

Though this method of is also represented by (MM/1: FCFS/D/D).

It is general queuing model in which the arrival and the service rate depend upon the length of the queue. Some persons desiring service may not join the queue. Since it is too long thus affecting the arrival rate. Similarly service rate is affected by the length of the queue.

3) MODEL III:

This model is represent by (MM/1): (FCFS/N/∞). It is essentially the same as model I expect the service discipline is SIRO instead of FCFS.

4) MODEL IV:

This model is represented by (MM/I: FCFS/N/∞). In this model the capacity of the system is limited or finite say N. so the number of arrivals cannot exceed N.

5) MODEL V:

This model is represented by (MM/1: FCFS/N/M). Is final population or limited source model. In this model the probability of an arrival depends upon the number of potential customers. Available to enter the system.

6) MODEL VI:

This model is represented by(M/M/C): (FCFS/∞/∞). This is same as model I expert that there are c services channels working in parallel.

7) MODEL VII:

This model is represented by (M/Ek/1): (FCFS/∞/∞).in this model instead of exponential service time, there is a erlang service time with k phases.

8) MODEL VIII:

This is represented by (M/M/1): (GO/M/M), where m n. It represents machine repair problem with single repairman. N is the total no. machines out of which m are breakdown and forming a queue. Go represents genuine service discipline.

9) MODEL IX:

This model is represented by (M/M/C): (GO/M/M), M ≤ N. it is the same model VIII. Expect that there are repairmen c < n.

10) MODEL X:

This is called power supply model.

(B) DETERMINISTIC MODEL

11) MODEL XI:

This model is represented by (D/D/Z); (FCFS/∞/∞).In this model inter arrival as well as service time are fixed and now with certainty. The model is therefore called deterministic model.

(C) MIXED QUEUING MODEL

12) MODEL XI:

This model is represented by (M/D/1); (FCFS/∞/∞).Here arrival rate is poison distributor while the service rate is deterministic or constant.

SEQUENCING MODEL

Assumptions in sequence problems

- Assumptions are usually made while dealing with sequencing problem
- Only one operation is carried out on a machine at a particular time
- Each operation once started must be completed
- An operation must be completed before is succeeding operation can start
- Only one machine of each type is available
- A job is processed as soon as possible but only in the order specified
- Processing times or independent of order of performing the operation
- The transportation time i.e., the time required to transport jobs from one machine to another is negligible.
- Jobs are completely known and are ready for processing when the period under consideration starts
- The costs of in process inventory for each job is same and negligibly small

Process of n jobs through one machine

The case when a number of jobs is to be processed on a single facility is quite common in actual practice.

Examples: a number of patients waiting for a doctor
Consider r is different jobs with known processing times require processing on a single machine.

N = no of different jobs
Ti= processing time of jobs i,
Wi=waiting time(before processing) for job i,
Fi= following time of job i=wi+ti,
Ci=completion time of job i,
Di=due date of job i,
Ii = lateness of job i =ci-di,
Ei = earliness of job i=di-ci,
Ti = tardiness of job i,
And Nt = number of tardy jobs.

The selection of the appropriate order in which customer may be served is called sequencing.

There are two types of sequencing problem.

1) In this type there are 'n' different jobs to be performed where job requires processing on some or all of 'm' different types of machine.
2) The second type of problem deals with jobs having number of machines and a list of tasks to be performed. Each rime a task is completed by a machine the next task to be performed on it got to be decided. Thus the list of tasks will change as fresh orders are received.

The various optimality criteria normally restored to are:

a) Minimizing total elapsed time.
b) Minimizing mean flow time
c) Minimizing idle time of machines.
d) Minimizing total tardiness
e) Minimizing number of tardy jobs
f) Minimizing in-process inventory cost
g) Minimizing the cost of being late.

LATENESS OF JOB

Lateness of job is defined as three differences between the actual completion time of the job and its due time. If the lateness is positive, it is termed as tardiness. Total tardiness is the sum of tardiness over the entire job in the set.

ASSUMTIONS IN SEQUECING PROBLEM

The following simplifying assumptions are usually made while dealing with sequencing problem.

1) Only one operation is carried out on machine at a particular time.
2) Each operating once started must be completed.
3) An operation must be before its succeeding operation can start.

4) Only one machine of each type is available.
5) A job is processed as soon as possible, but only in the order specified.
6) Processing times are independent of order of performing the operations.
7) The transportation time i.e. the required to transport jobs from one machine to another is negligible.
8) Jobs are completely known and are ready for processing when the period under consideration starts.
9) The cost of in-process inventory each job is same and negligibly small.

PROCESSING N JOBS THROUGH ONE MACHINE:

Consider a static job shop where an n different job with known processing times requires processing times requires processing on a single machine. The job shop is static in the sense in that any new job that arrives does not disturb the processing of these n jobs. So it is assumed that new job arrivals wait for being considered in the next batch of jobs after the processing of the current n jobs is completed.

Let
n = no. of different jobs,
ti = processing time of job.i
Wi = waiting time (before processing) for job I,
Fi = flow time of job I = Wi + ti,
Ci = completion of job i,
Di+due date of job i,
Li = lateness of job i=ci-di
Ei = earliness of job i=di-ci
Ti = tardiness of jobi,
And Nt = number of tardy jobs,

Slack time remaining (STR) rule

Slack time for a job is defined as the due date of the job minus its processing time.Sequencing the jobs in such a way that the jobs with the least slack time are picked up first for processing, followed by the one with the next smallest slack time and soon is called the slack time remaining rule (STR).

SOLUTION OF COMPLICATED SEQUENCING PROBLEMS :

Overlap:

Some times a job may involve the processing a number of similar items. Situation may arise wherein a few first items coming out of one operation may go in for second before remaining items in the lot could go in for the first operation.

Movement time:

Movement of jobs from one facility to another may take a considerable amount of time and it has to be incorporated in to the model

Rework

If one of the sequencing operation in inspection, the defective items may have to be send back to an operation performed earlier for reprocess, resulting in either a delay or splitting the job in to two lots.

Expediting:

If a particular job in urgently required, it may have to be given priority and be processed earlier.

Machine breakdown:

A machine may break down or the operator may be injuried or absent from work.

Material storage:

A material required for carrying out a particular operation may not be readily available.

Variable processing times:

In multi, shift plants, the time required to perform an operation may vary from shift to shift. These times and the cost associated with them may even be of probabilistic nature approach.

Shortest Processing Time (SPT Rule)

Sequencing the job in a way with least processing time is picked up first followed by the one with the next smallest processing time and so on is known as SPT. Sequencing the following objectives are:

Minimizing mean waiting time
Minimizing mean float time
Minimizing mean lateness
Minimizing mean number of the jobs waiting as in process inventory

Slack time remaining (STR rule):

Slack time for a job is defined as the due date of the minus its processing time sequencing the jobs in such a way that the jobs with their least slack time are picked up first for processing followed by the one with the next smallest slack time and so on is called the slack time remaining (STR rule).

SOLUTION OF COMPLICATED SEQUENCING PROBLEMS

OVERLAP: sometimes a job may involve the processing of a number of similar items
MOVEMENT TIME: Movement of jobs from one facility to another may take a considerable amount of time and it has to be incorporated into the model.
REWORK: If one of the sequencing operation in inspection, the defective items may have to be send back to an operation performed earlier for reprocess, resulting in either a delay or splitting the job in to two lots.
EXPEDITING: If a particular job in urgently required, it may have to be given priority and be processed earlier.
MACHINE BREAKDOWN: A machine may break down or the operator may be injuried or absent from work.

MATERIAL STORAGE: A material required for carrying out a particular operation may not be readily available.

VARIABLE PROCESSING TIMES:

In multi, shift plants, the time required to perform an operation may vary from shift to shift. These times and the cost associated with them may even be of probabilistic nature approach.

PROBLEMS

1. The time required to perform the jobs on two machines is given below, find the processing time and the total time required to turn out all the jobs.

Job	1	2	3	4	5	6	7
M1	8	10	10	6	12	1	3
M2	3	12	15	6	10	11	9

2. The time required to perform the jobs on two machines is given below, find the processing time and the total time required to turn out all the jobs.

Job	1	2	3	4	5	6
M1	8	10	15	6	12	1
M2	3	11	9	6	9	11

3. The time required to perform the jobs on two machines is given below, find the processing time and the total time required to turn out all the jobs.

Job	1	2	3	4	5	6
M1	5	7	2	6	3	4
M2	2	5	4	9	1	3

4. The time required to perform the jobs on two machines is given below, find the processing time and the total time required to turn out all the jobs:

Job	1	2	3	4	5	6
M1	7	8	9	6	5	4
M2	8	8	8	5	7	6

5. The time required to perform the jobs on two machines is given below, find the processing time and the total time required to turn out all the jobs.

Job	1	2	3	4	5	6
M1	5	9	4	7	8	6
M2	7	4	8	3	9	5

6. The time required to perform the jobs on two machines is given below, find the processing time and the total time required to turn out all the jobs.

Job	1	2	3	4	5
M1	5	7	2	6	3
M2	2	5	4	9	1

7. The time required to perform the jobs on two machines is given below, find the processing time and the total time required to turn out all the jobs.

Job	1	2	3	4	5	6
M1	6	5	7	9	8	10
M2	4	8	8	7	10	6

8. The time required to perform the jobs on two machines is given below, find the processing time and the total time required to turn out all the jobs.

Job	1	2	3	4	5	6
M1	2	3	4	6	7	9
M2	1	6	7	4	9	5

9. The time required to perform the jobs on two machines is given below, find the processing time and the total time required to turn out all the jobs.

Job	1	2	3	4	5
M1	2	7	4	9	8
M2	3	6	8	7	6

10. The time required to perform the jobs on two machines is given below, find the processing time and the total time required to turn out all the jobs.

Job	1	2	3	4	5	6
M1	2	4	6	8	7	4
M2	10	4	5	8	6	7

11. The time required to perform the jobs on two machines is given below, find the processing time and the total time required to turn out all the jobs.

Job	1	2	3	4	5	6
M1	10	12	18	11	18	19
M2	9	11	12	14	19	20

12. The time required to perform the jobs on two machines is given below, find the processing time and the total time required to turn out all the jobs.

Job	1	2	3	4	5	6
M1	12	11	10	8	9	13
M2	10	12	14	8	19	7

13. The time required to perform the jobs on three machines is given below, find the processing time and the total time required to turn out all the jobs.

Job	1	2	3	4	5	6	7
M1	3	8	7	4	9	8	7
M2	4	3	2	5	1	4	3
M3	6	7	5	11	5	6	12

14. The time required to perform the jobs on three machines is given below, find the processing time and the total time required to turn out all the jobs.

Job	A	B	C	D	E	F	G
M1	3	8	7	4	9	8	7
M2	4	3	2	5	1	4	3
M3	6	7	5	11	5	6	12

15. The time required to perform the jobs on three machines is given below, find the processing time and the total time required to turn out all the jobs.

Job	A	B	C	D	E	F	G
M1	13	12	11	10	9	8	7
M2	4	3	2	5	1	4	3
M3	12	13	11	11	5	6	12

16. The time required to perform the jobs on three machines is given below, find the processing time and the total time required to turn out all the jobs.

Job	A	B	C	D	E	F	G
M1	3	8	7	4	9	8	7
M2	4	3	2	5	1	4	3
M3	6	7	5	11	5	6	12

17. The time required to perform the jobs on three machines is given below, find the processing time and the total time required to turn out all the jobs.

Job	A	B	C	D	E	F	G
M1	3	8	7	4	9	8	7
M2	3	3	2	4	1	5	3
M3	6	7	5	11	5	7	11

18. The time required to perform the jobs on three machines is given below, find the processing time and the total time required to turn out all the jobs.

Job	A	B	C	D	E	F	G
M1	3	4	5	4	5	6	7
M2	4	5	6	7	6	8	7
M3	9	8	10	9	12	8	10

19. The time required to perform the jobs on four machines is given below, find the processing time and the total time required to turn out all the jobs.

Job	A	B	C	D	E	F	G
M1	3	8	7	4	9	8	7
M2	4	3	2	5	1	4	3
M3	6	7	5	4	5	6	4
M4	10	9	7	8	10	9	8

REPLACEMENT

There are two types of failures that are consider in replacement theory they are

*Gradual failure
*Sudden failure

Gradual Failure: The failure mechanism under this category is progressive, that is as the life of an item increase, its efficiency decreases resulting in

(i) increased expenditure for operating costs
(ii) decreased output of the machine
(iii) decrease in resale value of the machine

Such situation arises in items like cars, machines, trucks, automobile tyres, etc.

Sudden Failure: This class of failure is not applicable to items that deteriorates with time but to items that fail suddenly after a period of use. But this period is not a constant but may follow a probability distribution which may be progressive, retrogressive or random.

Progressive failure: Under this category, the probability of failure increases with the age for example such a thing occurs in electrical bulbs, automobile tyres, etc.

Retrogressive failure: There are items having greater probability of failure in the early life and the chance of failure decreases as age increases its expected life. These types of failure occurs in aircraft engines.

Random failure: There are items where in failure may occur randomly independent of the age of item. For example vacuum tubes in air-borne equipment have been shown to fails at a rate independent of the age of the tube.

Preventive Replacement: There are items which may from a part of large complete system. If any of the items fails, then the entire system breaks down. The sudden failure of such items may result in loss of production, idle inventory, labour and other such losses. If it is possible to predict as to when such item is going to fail then replacing such item before its failure is possible prevailing the sudden breakdown of the system. Such a case is known as preventive replacement.

Strategies of replacement: There are three types of replacement policies which follow sudden failure mechanism. They are

(i) Replacement of an item as when it fails.
(ii) Individual preventive replacement
(iii) Common preventive replacement.

Individual preventive replacement

According to this an item has to be replaced immediately after its failure or preventive maintenance is performed in it by knowing its optimum life. That is the item is replaced after its known optimum life period even though it may still survive.

Common preventive replacement: This strategy is applied for a system consisting of a group of items. Here the individually failed items are replaced as and when they fail and all the items are replaced after the optimal period. This situation is also called group replacement.

Next our interest will be to determine the age of replacement of such items. In case of items which deteriorate with time we consider the following two situations.

(i) Replacement of items whose maintenance cost increases with time and money value is not considered.

(ii) Replacement of items whose maintenance cost increase with time and money value changes with time

We shall now derive formula for replacement of items in the above two cases.

For a machine, the maintenance cost always in crease with time and a stage comes when the maintenance cost is so large. It is uneconomical to continue the item in service and hence we have to replace this item by a new one. Hence we have to determine the best are at which the replacement should take place.

PROBLEMS

21. For a machine, the following data are available:

Year	0	1	2	3	4	5	6
Cost of Spares (Rs.)	-	2000	4000	7000	10000	14000	16000
Salary of Maintenance Staff (Rs.)	-	12000	12000	14000	16000	20000	26000
Losses due to Break Down (Rs.)	-	6000	8000	7000	10000	12000	16000
Resale Value	120000	60000	30000	15000	8000	4000	4000

22. A company has 1000 bulbs and it costs Rs 5 to replace an individual bulb. If all bulbs are replaced simultaneously it would cost Rs. 1.35 per bulb. It is proposed to replace all bulbs at fixed intervals of time, whether or not they have burnt and continue replacing burnout as they fail. At what intervals the group replacement should be made?

Also prove that this optimal policy is superior to the straight forward policy of replacing each bulb as and when it fails. The mortality rates of the bulbs are as follows:

Week	**1**	**2**	**3**	**4**	**5**
% of failing at the end of each week	5	15	45	85	100

23. For a machine, the following data are available.

Year	0	1	2	3	4	5	6
Cost of Spares (Rs.)	-	200	400	700	1000	1400	1600
Salary of Maintenance Staff (Rs.)	-	1200	1200	1400	1600	2000	2600
Losses due to Break Down (Rs.)	-	600	800	700	1000	1200	1600
Resale Value	12000	6000	3000	1500	800	400	400

24. A company has purchased a mini computer MINI COMP. It costs Rupees 2.5 lakhs and running and maintenance costs are Rs. 1,20,000 for each of the first five years, increasing by Rs. 20,000 per year in the sixth and subsequent years. If money's worth is 10% per year when should the company replace the computer.

25. A company has 1000 bulbs and it costs Rs 4 to replace an individual bulb. If all bulbs are replaced simultaneously it would cost Rs. 1.25 per bulb. It is proposed to replace all bulbs at fixed intervals of time, whether or not they have burnt and continue replacing burnout as they fail. At what intervals the group replacement should be made? Also prove that this optimal policy is superior to the straight forward policy of replacing each bulb as and when it fails. The mortality rates of the bulbs are as follows:

Week	**1**	**2**	**3**	**4**	**5**
% of failing at the end of each week	5	25	65	85	100

26. The data collected in running a machine the cost of which is Rs 60,000 are given below

Year	1	2	3	4	5
Real Value	42000	20000	18400	14400	9650
Cost of Spares (Rs.)	4000	4270	4880	5700	6800
Cost of Labour (Rs.)	14000	16000	18000	21000	25000

27. A company has purchased a mini computer MINI COMP. It costs Rupees 2.5 lakhs and running and maintenance costs are Rs. 1,00,000 for each of the first five years, increasing by Rs. 25,000 per year in the sixth and subsequent years. If money's worth is 10% per year when should the company replace the computer.

28. For a machine, the following data are available:

Year	0	1	2	3	4	5	6
Cost of Spares (Rs.)	-	100	200	350	500	700	800
Salary of Maintenance Staff (Rs.)	-	600	600	700	800	1000	1300
Losses due to Break Down (Rs.)	-	300	400	350	500	600	800
Resale Value	6000	3000	1500	750	400	200	200

29. The data collected in running a machine the cost of which is Rs 30,000 are given below

Year	1	2	3	4

Real Value	20000	18400	14400	9650
Cost of Spares (Rs.)	4270	4880	5700	6800
Cost of Labour (Rs.)	16000	18000	21000	25000

30. A company has 800 bulbs and it costs Rs 5 to replace an individual bulb. If all bulbs are replaced simultaneously it would cost Rs. 1.50 per bulb. It is proposed to replace all bulbs at fixed intervals of time, whether or not they have burnt and continue replacing burnout as they fail. At what intervals the group replacement should be made? Also prove that this optimal policy is superior to the straight forward policy of replacing each bulb as and when it fails. The mortality rates of the bulbs are as follows:

Week	**1**	**2**	**3**	**4**
% of failing at the end of each week	5	35	75	100

31. A manufacturer is offered two machines A and B. A is priced at Rs.5,000 and running costs are estimated at Rs. 800 for each of the first 5 years increasing by Rs. 200 in the sixth and subsequent years. Machine B, which has the same capacity as A, costs Rs. 2,500 but with running costs of Rs. 1,200 per year for the first six years and increasing by Rs. 200 per year thereafter. If money is worth 10% per year which machine should be purchased?

32. A company has 800 bulbs and it costs Rs 6 to replace an individual bulb. If all bulbs are replaced simultaneously it would cost Rs. 1.50 per bulb. It is proposed to replace all bulbs at fixed intervals of time, whether or not they have burnt and continue replacing burnout as they fail. At what intervals the group replacement should be made? Also prove that this optimal policy is superior to the straight forward policy of replacing each bulb as and when it fails. The mortality rates of the bulbs are as follows:

Week	**1**	**2**	**3**	**4**
% of failing at the end of each week	5	35	75	100

33. For a machine, the following data are available:

Year	0	1	2	3	4	5	6
Cost of Spares (Rs.)	-	400	800	1200	1400	2800	3200
Salary of Maintenance Staff (Rs.)	-	2400	2400	2800	3400	4400	5400
Losses due to Break Down (Rs.)	-	1200	1600	1400	2000	2500	3500
Resale Value	2200	11000	1500	750	375	200	100

34. A company has 1000 resistors and it costs Rs 5.50 to replace an individual resistors. If all resistors are replaced simultaneously it would cost Rs. 1.50 per resistors. It is proposed to replace all resistors at fixed intervals of time, whether or not they have burnt and continue replacing burnout as they fail. At what intervals the group replacement should be made? Also prove that this optimal policy is superior to the straight forward policy of replacing each resistor as and when it fails. The probability of resistors left out after each month are as follows:

Week	**1**	**2**	**3**	**4**	**5**	**6**
% of failing at the end of each week	.85	.75	.50	.35	.15	0

35. For a machine, the following data are available.

Year	0	1	2	3	4	5	6
Cost of Spares (Rs.)	-	200	400	700	1000	1400	1600
Salary of Maintenance Staff (Rs.)	-	1200	1200	1400	1600	2000	2600
Losses due to Break Down (Rs.)	-	600	800	700	1000	1200	1600

Resale Value	12000	6000	3000	1500	800	400	400

36. A company has 600 resistors and it costs Rs 3 to replace an individual resistors. If all resistors are replaced simultaneously it would cost Rs. 0.85 per resistors. It is proposed to replace all resistors at fixed intervals of time, whether or not they have burnt and continue replacing burnout as they fail. At what intervals the group replacement should be made? Also prove that this optimal policy is superior to the straight forward policy of replacing each resistor as and when it fails. The percentage of resistors left out after each month are as follows:

Week	**1**	**2**	**3**	**4**	**5**
% of failing at the end of each week	90	75	55	25	0

37. A company has 1000 resistors and it costs Rs 50 to replace an individual resistors. If all resistors are replaced simultaneously it would cost Rs. 12.5 per resistors. It is proposed to replace all resistors at fixed intervals of time, whether or not they have burnt and continue replacing burnout as they fail. At what intervals the group replacement should be made? Also prove that this optimal policy is superior to the straight forward policy of replacing each resistor as and when it fails. The probability of resistors left out after each month are as follows:

Week	**1**	**2**	**3**	**4**	**5**	**6**
% of failing at the end of each week	.85	.75	.50	.35	.15	0

38. The data collected in running a machine the cost of which is Rs 5,000 are given below

Year	0	1	2	3	4	5	6	7
Resale Value	3500	2500	1700	1200	800	500	500	500
Running Cost (Rs.)	1500	1600	1800	2100	2500	2900	3400	4000

39. A company has purchased a crane. It costs Rupees 2 lakhs and running and maintenance costs are Rs. 10,000 for each of the first five years, increasing by Rs. 25,000 per year in the sixth and subsequent years. If money's worth is 14% per year when should the company replace the computer.
40. A company has 800 resistors and it costs Rs 50 to replace an individual resistors. If all resistors are replaced simultaneously it would cost Rs. 12.5 per resistors. It is proposed to replace all resistors at fixed intervals of time, whether or not they have burnt and continue replacing burnout as they fail. At what intervals the group replacement should be made? Also prove that this optimal policy is superior to the straight forward policy of replacing each resistor as and when it fails. The probability of resistors left out after each month are as follows:

Week	**1**	**2**	**3**	**4**	**5**	**6**
% of failing at the end of each week	.85	.75	.50	.35	.15	0

41. The cost of a machine is Rs. 6100 and its scrap value is Rs. 100. The maintenance costs found from experience are as follows:

Year	1	2	3	4	5	6	7	8
Maintenance Costs (Rs.)	100	250	400	600	900	1200	1600	2000

42. A company has 500 bulbs and it costs Rs 4.5 to replace an individual bulb. If all bulbs are replaced simultaneously it would cost Rs. 1.20 per bulb. It is proposed to replace all bulbs at fixed intervals of time, whether or not they have burnt and continue replacing burnout as they fail. At what intervals the group replacement should be made? Also prove that this optimal policy is superior to the straight forward policy of replacing each bulb as and when it fails. The mortality rates of the bulbs are as follows:

Week	**1**	**2**	**3**	**4**	**5**

% of failing at the end of each week	.05	.15	.45	.85	1.00

43. A manufacturer is offered two machines A and B. A is priced at Rs.50,000 and running costs are estimated at Rs. 8000 for each of the first 5 years increasing by Rs. 2000 in the sixth and subsequent years. Machine B, which has the same capacity as A, costs Rs. 25000 but with running costs of Rs. 2,400 per year for the first six years and increasing by Rs. 400 per year thereafter. If money is worth 12% per year which machine should be purchased?
44. A company has 800 resistors and it costs Rs 5 to replace an individual resistors. If all resistors are replaced simultaneously it would cost Rs. 1.35 per resistors. It is proposed to replace all resistors at fixed intervals of time, whether or not they have burnt and continue replacing burnout as they fail. At what intervals the group replacement should be made? Also prove that this optimal policy is superior to the straight forward policy of replacing each resistor as and when it fails. The probability of resistors left out after each month are as follows:

Week	**1**	**2**	**3**	**4**	**5**	**6**
% of failing at the end of each week	.80	.70	.40	.25	.15	0

45. For a machine, the following data are available:

Year	0	1	2	3	4	5	6
Cost of Spares (Rs.)	-	200	400	700	1000	1400	1600
Salary of Maintenance Staff (Rs.)	-	1200	1200	1400	1600	2000	2600
Losses due to Break Down (Rs.)	-	600	800	700	1000	1200	1600
Scrap Value	18000	9000	1500	800	600	300	150

46. A company has 5000 bulbs and it costs Rs 5 to replace an individual bulb. If all bulbs are replaced simultaneously it would cost Rs. 1.5 per bulb. It is proposed to replace all bulbs at fixed intervals of time, whether or not they have burnt and continue replacing burnout as they fail. At what intervals the group replacement should be made? Also prove that this optimal policy is superior to the straight forward policy of replacing each bulb as and when it fails. The mortality rates of the bulbs are as follows:

Month	**1**	**2**	**3**	**4**	**5**	**6**
% of failing at the end of each week	13	27	53	79	88	100

47. For a machine, which costs Rs. 24000, the following data are available:

Year	1	2	3	4	5	6
Cost of Spares (Rs.)	500	1000	1400	2000	2800	3200
Salary of Maintenance Staff (Rs.)	2400	2400	3000	3500	4200	5500
Losses due to Break Down (Rs.)	900	1700	1500	2200	2500	3200
Resale Value	23000	20000	18000	12000	8000	4000

48. A decorative series lamp set circuit contains 10,000 bulbs. When any bulb fails, it is replaced. The cost of replacing a bulb individually is Re. 1 only. If all the bulbs are replaced at the same time, the cost per bulb would be reduced to 35 paise. The present surviving say s(t) at the end of month t and p(t), the probabilities of failure during the month t are given below; find the optimal replacement plan.

Week	**0**	**1**	**2**	**3**	**4**	**5**	**6**
s(t)	100	97	90	70	30	15	0
p(t)	-	0.03	0.07	0.20	0.40	0.15	0.15

49. A decorative series lamp set circuit contains 50,000 bulbs. When any bulb fails, it is replaced. The cost of replacing a bulb individually is 80 paise only. If all the bulbs are replaced at the same time, the cost per bulb would be reduced to 25 paise. The present surviving say s(t) at the end of month t and p(t), the probabilities of failure during the month t are given below; find the optimal replacement plan.

Week	**0**	**1**	**2**	**3**	**4**	**5**	**6**
s(t)	100	95	85	70	30	15	0
p(t)	-	0.05	0.10	0.20	0.40	0.15	0.15

50. A company has purchased a car. It costs Rupees 3.5 lakhs and running and maintenance costs are Rs. 10,000 for each of the first five years, increasing by Rs. 20,000 per year in the fifth and subsequent years. If money's worth is 8% per year when should the company replace the computer.

Unit – V
GAME THEORY

1.What is games theory?

In business management and commerce we come across situations involving conflicting interests. To tackle such situations a special discipline called Game Theory has been developed. In game theory we consider two or more persons called players (competitor) with different objective, each of whose actions influence the outcomes of the other.

2.What is Maximin or Minimax Criterion?

Each player wants to maximize his minimum expected gain or minimize his maximum expected loss. This criterion is referred to as minimax or maximin criterion.

3.Short notes on

Player: A Player is called a competitor.

Strategy: A strategy for a player is a plan, which specifies his action for every possible action of his opponent.

Finite Game: If the number of strategies of the players is finite then the game is said to be a finite game.

Infinite Game: If at least one of the players has infinite number of strategies, the game is said to be infinite.

Pay off: The different strategies adopted by the players will result in different outcomes of pay off. This will result in a gain or loss or draw for the players. In simple the winnings and losses of both the players represented in a matrix form is known as pay off matrix.

Two Player Zero Sum Game: A game with two players wherein one person's gain is the loss of the other is called two person zero sum game theory.

Game Matrix: The pay offs corresponding to various strategies of the players are represented in a matrix is called the game matrix.

Pure Strategy: If the best strategy for each player is to play one particular strategy throughout the game then it is called Pure Strategy.

Mixed Strategy: If the optimal plan for each player is to choose different strategies at different situations then it is called Mixed Strategy game.

Value of the Game: It is the maximum guaranteed gain to the player B. When the maximin value = the minimax value = v, then v is called the value of the game.

Optimal solution for two person zero sum game: The optimal solution to a two person zero sum game is obtained by minimax – maximin principle. According to this the player A represented by the row selects the strategy which maximizes his minimum gain, the minimum being taken over all the strategies of the player B. In a similar way, player B selects his strategy, which minimizes his maximum loss.

Saddle Point: The position in the pay off matrix corresponding to the optimal strategies is called Saddle Point.

Optimal Strategies: The pure strategies corresponding to the value of the game are optimal strategies.

4. Write short notes on the rules of applications of game theory?
The basic rules for the application of games theory are described below:
Rule 1: For the application of the game theory in to the problem we have to look for a pure strategy i.e. saddle point in the problem. If we get the saddle point then it's the saddle point for the problem.

Rule 2: If no pure strategies exist, then the next step is to eliminate certain strategies i.e. row and columns of the problem using the dominance rule. We have to compare two rows and we have to remove the dominated row and if any two columns are compared then the dominating column is to be removed.

Rule3: In case where there is no saddle point and dominance has been used to reduce the game matrix, players will resort to mixed strategies. Arithmetic and algebraic methods are used for finding the optimum strategies as well as the value of the game.

5. For what type of business problems might the games theory be helpful?

The games theory is a mathematical theory that deals with the general features of competitive situations. This theory is helpful when two or more individuals or organizations with conflicting objectives

try to make decisions. In such situations, decisions made by one decision maker affect the decision made upon the decision of all the parties. This games theory is applicable to a wide variety of situation such as two firms struggling to maintain their market shares, launching advertisement campaigns by companies marketing competing products, negotiations between organization and unions etc.

6. Describe the role of theory of games for scientific decision-making.

a. It helps us to choose the best among the "n" number of organization in maintaining the market share.

b. It helps to know all the possible choice available to all the organization in the market and to take good decision.
c. The choices are assumed to be made simultaneously so that no participant knows the choices made by others until how has decided his own.
d. The decision made by any of the organization is not based or depends on their own actions but also those of others.
e. The organizations make individuals decisions without direct communication.

7. State the four properties, which a competitive situation should have if it is to be called a competitive game.

A competitive game has the following characteristics:

a) There are finite number of participants or competitors, if the number of participants is 2, the game is called two player game, for no greater than 2, it is called n- person game.
b) Each participant has a list of finite number of possible courses of action. This list may not be same for each participant.
c) Each participant knows all the possible choices available to others but does not know which of them is going to be chosen by them.
d) Every combination of courses of action determines an outcome, which results in gains to the participants. The gain may be positive, negative or zero. Negative gain is called a loss.
e) The gain of a participant depends not only on his own action but also those of others.
f) The players make individual decisions without direct communication.

8. How is the concept of dominance used in simplifying the solution of a rectangular game?

If no pure strategies exist the next step is to eliminate certain strategies (rows and/or columns) by dominance. The resulting game can be solved very easily using some algebraic method or arithmetic method.

The dominance rule for column is: If all the values of a column are equal or greater than the corresponding elements of another column then the dominating (bigger) column must be removed from the matrix.

The dominance rule for row is: If all the values of a row are equal or smaller than the corresponding elements of another row then the dominated (smaller) row must be removed from the matrix.

10. Explain The Minimax And Maximin Principle

This principle is used for the selection of optimal strategies by two players. Consider two players A and B. A is a player who wishes to maximize his gain while player B wishes minimize his losses. Since A would like to maximize his minimum gain, we obtain for player A, the value called maximin value and the corresponding strategy is called the maximin strategy.

On the other hand, since player wishes to maximize his losses, a value called the minimax value which is the minimum of the maximum losses is found. A corresponding strategy is called the minimax strategy. When this two or equal (maximin value = minimax value), the corresponding strategy are called optimal strategy and the game is said to have a saddle point.

11. Dominance Rule or Dominance Property

If no pure strategies exit, a next step is to eliminate certain strategies(row and column) by dominance. The result in game can be solved for some mixed strategy.

PROBLEMS

1. Solve the following game:

Player A	Player B			
	Sa	Sb	Sc	Sd
S1	-3	5	1	2
S2	-2	3	1	2
S3	2	1	3	4
S4	4	2	4	4

2. Solve the following game:

Player A	Player B			
	Sa	Sb	Sc	Sd
S1	5	4	-2	1
S2	7	-6	3	6
S3	12	8	10	9
S4	6	8	-9	14

3. Solve the following game:

Player A	Player B		
	Sa	Sb	Sc
S1	1	3	6
S2	2	3	3
S3	6	2	7

4. Solve the following game:

Player A	Player B			
	Sa	Sb	Sc	Sd
S1	10	8	-11	-2
S2	14	6	-5	5
S3	9	7	5	-4
S4	15	4	-3	3

5. Solve the following game:

Player A	Player B			
	Sa	Sb	Sc	Sd
S1	7	5	-3	3
S2	2	5	-1	4
S3	5	6	2	4
S4	1	4	-2	3

6. Solve the following game:

Player A	Player B			
	Sa	Sb	Sc	Sd
S1	4	5	7	3
S2	9	10	8	9
S3	2	3	5	6
S4	1	-2	0	4

7. A and B play the game as follows: They simultaneously and independently write one of the three numbers 1, 2 and 3. If the sum of the numbers is even. B pays to A their sum in rupees. If it is odd. A pays the sum to B in rupees. Form the matrix of the game for A and solve it.

8. Solve the following game graphically:

Player A	Player B				
	B1	B2	B3	B4	B5
A1	2	-2	3	7	6
A2	6	5	1	4	0

9. Solve the following game:

Player A	Player B			
	Sa	Sb	Sc	Sd
S1	7	5	-3	3
S2	1	2	2	-1
S3	5	6	2	4
S4	1	4	-2	3

10. Solve the following game graphically:

Player A	Player B

	B1	B2
A1	-6	7
A2	4	-5
A3	-1	-2
A4	-2	5
A5	7	-6

11. Solve the following game:

Player A	Player B				
	B1	B2	B3	B4	B5
A1	8	10	13	16	9
A2	7	12	6	15	10
A3	9	18	9	13	25
A4	4	9	8	20	6

12. Solve the following game graphically:

Player A	Player B				
	B1	B2	B3	B4	B5
A1	2	-2	3	7	6
A2	6	5	1	4	0

13. Solve the following game:

Player A	Player B			
	Sa	Sb	Sc	Sd
S1	7	5	-3	3
S2	1	2	2	-1
S3	5	6	2	4
S4	1	4	-2	3

14. Solve the following game graphically:

Player A	Player B				
	B1	B2	B3	B4	B5
A1	2	-2	3	7	6
A2	6	5	1	4	0

15. Two firms are competing for business. Whenever Firm A gains firm B loses. The table shows advertising strategies of both firms and the utilities to Firm A for various market shares in percentage. (Assuming this to the zero sum game)?

Firm A	Firm B		
	Press	Radio	TV
Press	60	45	40
Radio	75	75	60
TV	80	60	70

16. Solve the following game graphically:

Player A	Player B				
	B1	B2	B3	B4	B5
A1	2	-2	3	7	6
A2	6	5	1	4	0

17. Two persons A and B play the following game. A has a bag containing three coins, one worth one unit, one 3 units and the rest worth 6 units. A takes one coin from the bag and before it is exposed, B guesses what it is. If B is right he takes the coin and if he is wrong he gives to A a coin of same worth. Is this a fair game? What is the value of the game to A? What are A's and B's optimal strategies?

18. Solve the following game graphically:

Player A	Player B	
	B1	B2
A1	-6	7
A2	4	-5
A3	-1	-2
A4	-2	5
A5	7	-6

19. A student must decide how to study for a final examination in history. He studies differently for True – False, Multiple Choice and Essay exams and he does not know which type his exam will be. The student thinks that if he studies for a true – false test he will score 85 on a true – false test, 80 on a multiple choice test and 75 on an essay test. If he studies for a multiple choice test, he expects to score 85 on a true – false test, 90 on a multiple choice test and 85 on essay test. If he studies for essay test he expects to score 80 on a true or false test, 90 on a multiple choice test and 90 on an essay test. To maximize his minimum expected score for what type of the test should the student study?

20. Solve the following game graphically:

Player A	Player B				
	B1	B2	B3	B4	B5
A1	2	-2	3	7	6
A2	6	5	1	4	0